Exposure for Collaborative Detection Using Mobile Sensor Networks

Tai-Lin Chin[1], Parameswaran Ramanathan[1], Kewal K. Saluja[1], Kuang-Ching Wang[2]
[1]Department of Electrical and Computer Engineering, University of Wisconsin-Madison
[2]Department of Electrical and Computer Engineering, Clemson University
{tchin,parmesh,saluja}@ece.wisc.edu, kwang@clemson.edu

***Abstract*— Sensor networks possess the inherent potential to detect the presence of a target in a monitored region. Although a stationary sensor network is often adequate to meet application requirements, it is not suited to many situations, for example, a huge number of nodes are required to monitor a large region. In such situations, mobile sensor networks can be used to resolve the communication and sensing coverage problems. This paper addresses the problem of detecting a target using mobile sensor networks. One of the fundamental issues in target detection problems is *exposure*, which measures how the region is covered by the sensor network. While traditional studies focus on stationary sensor networks, this paper formally defines and evaluates exposure in mobile sensor networks with the presence of obstacles and noise. To conform with practical situations, detection is conducted without presuming the target's activities and moving directions. As there is no fixed layout of node positions, a time expansion technique is developed to evaluate exposure. Since determining exposure can be computationally expensive, algorithms to calculate the upper and lower bounds on exposure are developed. Simulation results are also presented to illustrate the effectiveness of the algorithms.**

I. Introduction

Sensor networks, capable of sensing the environment, exchanging data, processing information, and responding queries, provide an effective connection between computation systems and the physical world. Such networks collect data from the monitored environment and extract useful information to enable a variety of purposes such as battlefield surveillance [1], [2], and environmental monitoring [3], [4]. As advances in technology progress, one can envision that mobility will become readily available to nodes for handling more sophisticated sensing tasks in the near future.

In stationary sensor networks, network connectivity and sensing coverage can cause scalability problems in some circumstances. For example, a conventional approach for detecting an intruder in battlefield is to deploy a set of stationary nodes in the monitored region. However, if the monitored region is relatively large to either the sensing or the communication range of the nodes, a huge number of nodes are required to be deployed. Typically, the number of nodes to ensure connectivity with a sufficiently high probability in a random deployment is $O(\frac{L^2 \log L}{R^2})$, where L^2 is the area of the region and R is the communication range [5]. Similar effect would occur with respect to the sensing range. In such situations, mobile nodes can be an attractive alternative. Nodes can be carried by security guards or embedded with unmanned vehicles capable of automatic navigation to sample the environment. Thus, the sensing task can be performed effectively and inexpensively with fewer nodes.

For stationary sensor networks, *exposure*, defined as the least probability of detecting a target traversing through the monitored region [1], has been used in literature for assessing the quality of the sensor deployment. The higher the exposure, the better the deployment has been. As exposure is associated with the worst case path along which the target is most difficult to detect, the analysis allows one to diagnose the weakness of a sensor network and potentially improve it. Although exposure has been studied extensively in stationary sensor networks, it is never defined and neither directly applicable to mobile sensor networks. Since both the nodes and the potential target may change locations at any time in mobile sensor networks, the measurements of the target are determined by the instantaneous locations of the target as well as all active mobile nodes at each time instant. Apparently, the conventional exposure computation methods cannot capture this property and does not accommodate the sequential movements of the nodes.

This paper proposes a new collaborative sensing architecture using mobile nodes and formally defines the exposure under this sensing architecture. For accurately reflecting the quality of the mobile sensing mechanism by exposure, it is essential to identify the following three aspects of the network:

- Mobile sensing policy: We assume the mobile nodes patrol a monitored region following a set of given routes. The mobile nodes sample the environment at carefully selected sensing positions along the routes and collaboratively make consensus decisions on the presence/absence of the target.
- Target traversal strategy: A target is assumed to enter the monitored region, potentially stay for a certain time, and finally leave the region. Unlike previous work in literature [1], [6], we assume no prior knowledge of the target's traversal pattern. A target can enter and leave the region via any boundaries and performs idling or traversing activities for any period of time until it leaves. It is also important to recognize that the start time of traversal, which has no effect on the exposure in a stationary network, would actually affect the exposure in a mobile network. A target can potentially decide its entrance and departure time intelligently to reduce the probability of being detected, if they can discover or speculate the routes and sensing schedules of mobile nodes.
- Obstacles: With the presence of obstacles, the propagation of target signals can be obstructed from the sensors.

Intelligent target may take the advantage of obstacles to avoid directly exposing to the nodes. However, properly planning the routes for mobile nodes can also alleviate weak spots in the region.

These are the key characteristics of mobile sensing that renders the conventional exposure definition inadequate. The different interaction between sensor nodes and targets also indicates the clear need of an accurate exposure definition and an efficient assessment mechanism to guide the design of mobile nodes' routes and sensing schedule.

Searching the exposure path involves finding an appropriate time for the target entering the region and a path through which the target traverses. In this paper, a time expansion approach is proposed to measure the probability of detecting a target along a target path. We assume a minimal presence duration T is required for a target to accomplish a special task. To determine exposure can be computationally expensive since all possible target paths and motion patterns which result in the target presence of *at least* T time instants are included in the search space. Instead, by identifying and proving the upper and lower bounds on exposure, algorithms are devised to efficiently compute the exposure bounds for any number of mobile nodes with various routes and sensing schedules. With simulations, we demonstrate exposure analysis and illustrate the identified vulnerabilities in the region.

The rest of the paper is organized as follows. In Section II, related work in close contexts are reviewed. Section III describes the sensing architecture and detection model. In Section IV, we formally define exposure in mobile sensor network and present proofs and algorithms to calculate the upper and lower bounds on exposure. Section V summarizes the experimental results. The paper concludes in Section VI.

II. Related Work

Exposure and coverage of stationary sensor networks have been studied extensively in various forms [1], [6]–[8]. In [6], exposure of a path is defined as the integral of signal intensity measured by sensors along a target path. The paper proposes a centralized approach to identify the minimum exposure path in stationary sensor networks. In contrast, localized algorithms for searching the minimum exposure path are proposed in [8] using the same definition of exposure as in [6]. Instead of using signal intensity as an intermediate indicator of detection likelihood, a probabilistic model accounting for noise is proposed in [1]. The trade-off between exposure and false alarm probability is also studied. This model has been adopted to study sensor deployment strategies [7] and exposure of targets traversing at varying speeds [9]. A number of studies also used sensor network coverage to design deployment strategies. In [10], an integer programming based algorithm is developed to minimize the cost of completely covering the region of interest. To maximize sensing coverage of a network, a strategy based on attractive/repulsive virtual forces between nodes is proposed in [11], and a greedy algorithm that step-by-step places a new node at the most vulnerable point in a region is proposed in [12]. While all of these work studied exposure or coverage in stationary sensor networks, we focus on evaluating exposure in mobile sensor networks.

Recently, a few studies have started to explore the use of mobile nodes with different movement strategies to enhance operations in sensor networks as well as ad hoc communication networks. In [13], power efficiency of stationary sensors for data collection is improved by using a number of random roaming mobile entities called data MULEs. Experiments on exploiting random mobility such as mounting sensors on animals are studied in [3], [4]. Besides, controlled mobile nodes have also been used in many cases. In [14], mobile nodes exercise distributed mobility control to minimize their communication power. In [15], controlled mobile nodes called message ferries are routinely go on circuit to pick up and deliver data. In [16], mobile routers are used to help conserve energy at stationary nodes. These work have been focused on the data transport capability of mobile nodes, in contrast to our focus on collaborative sensing and detection with mobile nodes.

Earlier experiences with collaborative operation of networked mobile agents can be found in robotics studies such as pursuit-evasion games [17], [18]. Despite their similarity with a distributed detection problem, their objectives are rather different. In [19], a team of pursuers moving on the arcs of a graph is deployed to capture a fugitive who has complete knowledge of the pursuers' locations. The problem is to find a smallest number of pursuers that guarantee to capture the fugitive. In [20], the pursuers utilize their recent sensory inputs to adaptively determine the new movement directions of finding the evader. Overall, the objective of these studies is to capture a target whose existence is known. Whereas, the detection problem addressed in this paper is to detect the presence of a target without prior knowledge of its existence.

III. Sensing and detection models

In this section, the sensing architecture of the mobile sensor network is first described. The sensing architecture explains how the mobile nodes collaborate to detect an unauthorized target in the region. We then detail the target energy model and the detection technique proposed in [1], [2], such that the method to calculate exposure in mobile sensor networks can be developed.

A. Sensing Architecture

Consider each mobile node samples the environment for a target while patrolling along a given route. A target is an intruder entering the region from any boundary of the region and conducting unauthorized tasks such as spying an asset or collecting useful statistics, which typically require the target staying in the region for a certain amount of time. During this period of time, the target can be either idling at a certain position or traversing from one position to another. The target then leaves the region after the task is finished.

A route for a mobile node consists of an ordered sequence of sensing positions where the node takes measurements of the environments. Mobile nodes patrol the routes and report detection measurements to a fusion center periodically through wireless communications. The measurements are fused in the fusion center to arrive at a consensus decision on whether the

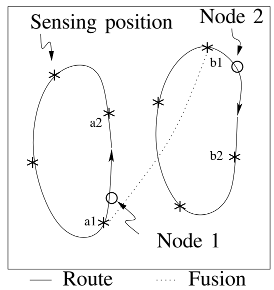


Fig. 1. A mobile sensor network with two nodes.

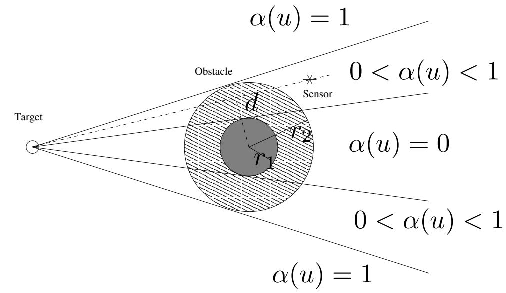


Fig. 2. The energy distortion factor $\alpha(u)$.

target is present. Fig. 1 shows an example of the routes and the sensing positions for two mobile nodes. In the example, node 1 and node 2 take measurements at position $a1$ and $b1$ respectively at a certain time instant. They report the measurements to the fusion center and then go to the next sensing positions. At the next time instant, node 1 and node 2 report the measurements again at $a2$ and $b2$ respectively. In a large region, the measurements can be transferred through satellites or pre-deployed base stations. This sensing architecture provides periodic detection results so the surveillance system can react to the current condition in the region.

B. Target Model

Consider n mobile nodes patrolling a region along a set of given routes. Let s_i, $1 \le i \le n$, be a snapshot of the sensing positions at a certain time instant and u be the target location. The target activities are assumed to emit signals which can be detected by the nodes. In general, the energy of the signals decays exponentially with distance. If the decay coefficient is k, the signal energy measured by the node at s_i is given as

$$S_i(u) = \begin{cases} \frac{\alpha_i(u)K}{|u-s_i|^k} & \text{if } |u - s_i| > c \\ K & \text{otherwise,} \end{cases} \tag{1}$$

where K is the energy emitted by the target, $\alpha_i(u)$ is the energy distortion factor due to obstacles, $|u - s_i|$ is the Euclidean distance between the target and the node, and c is the constant energy range which is a function of the size of the target. Normally, the decay factor k is between 2.0 to 5.0 [21].

Obstacles in the region can have different impacts on the energy level measured by sensor nodes. Some obstacles can absorb, while others can reflect, all or part of the signal energy. In this paper, the impact is characterized by the energy distortion factor $\alpha_i(u)$. If reflection of energy is negligible, the energy distortion factor used in this paper can be obtained as shown in Fig. 2. Obstacles are characterized by two radii, r_1 and r_2. Consider the two cones formed from the target to the inner circle and the outer circle. A node which lies in the inner cone beyond the obstacle has distortion factor $\alpha_i(u)$ equal to zero. A node which lies outside the outer cone has $\alpha_i(u)$ equal to one. Between the two cones, $\alpha_i(u)$ increases linearly from zero to one with the distance $d - r_1$, where d is the distance from the center of the obstacle to the line joining the target and the node. When there are multiple obstacles between the target and the node, the distortion factor is assumed to be the product of the distortion effect of each obstacle. With this formulation, a fully absorbing obstacle can be modeled by $r_1 = r_2$, and an only partially absorbing obstacle can be modeled by $r_1 = 0$. This paper only considers round obstacles, but the formulation can be extended for obstacles with other shapes in a similar manner.

Energy measurements of sensor nodes are inherently corrupted by noise. Denote the noise energy detected by node i as N_i^2 during a particular measurement. Then, the total energy measured at node i when the target is at location u is

$$E_i(u) = S_i(u) + N_i^2,$$

where $S_i(u)$ is the target energy from Equation (1). In this paper, we assume that noise is Additive White Gaussian Noise (AWGN) with mean zero and variance one, and is independent at each node. Thus, the noise energy N_i^2 is a Chi-square random variable with one degree of freedom.

C. Detection

The probability of detecting a target at a certain time instant is based on the value fusion approach proposed in [2]. In value fusion, the nodes collaborate to detect the target by sending the energy measurements to a manager node. The manager node sums up the received data and compares with a threshold η to arrive at a consensus decision on whether the target is present. Therefore, the probability of detecting a target at position u can be expressed as

$$d(u) = Prob\left(\sum_{i=1}^{n} N_i^2 + \sum_{i=1}^{n} S_i(u) > \eta\right) \tag{2}$$

$$= 1 - F_{\chi_n^2}\left(\eta - \sum_{i=1}^{n} S_i(u)\right) \tag{3}$$

where $F_{\chi_n^2}$ is the cumulative distribution function of Chi-square with n degrees of freedom. Note that the sum of n Chi-square random variable with one degree of freedom is a Chi-square random variable with n degrees of freedom. The method of how to choose the threshold η is presented in the following section.

IV. Exposure of mobile sensor networks

In this section, we formally define the exposure for the proposed collaborative sensing architecture. As nodes in such networks do not have fixed positions over time and the entrance and leave time of the target may affect the detection probability, we propose a time expansion approach to compute exposure. Efficient algorithms for determining the upper and lower bounds of the exposure are also developed.

A. Definition

Consider a target entering the monitored region for at least T time units. In general, the path traversed by the target can be fairly arbitrary in shape. To facilitate the computation, the monitored region is divided into a $N \times N$ grid. The granularity of the grid trades off the computation load with accuracy. The target is assumed to be able to stay at a grid point or traverse from one grid point to another in the region. Thus, the target location can be represented as (x, y), where x and y are integers in the range 0 to $N-1$. A point (x, y) is on the boundary if either x or y is either 0 or $N-1$. Using the grid coordinates, a target traversal can be formally defined as follows.

Definition 1. A *target traversal* with length l time units starting at time t is a sequence of triples of the form (x_0, y_0, t), $(x_1, y_1, t+1)$, ... $(x_l, y_l, t+l)$ such that

1) (x_0, y_0) is a point on the boundary
2) (x_l, y_l) is a point on the boundary,
3) For $0 \le i \le l$, $0 \le x_i, y_i \le N-1$, and
4) For $0 \le i \le l-1$, (x_{i+1}, y_{i+1}) is either (x_i, y_i) or (x_i+1, y_i) or (x_i-1, y_i) or (x_i, y_i-1) or (x_i, y_i+1).

The semantics of the above path is that the target is at location (x_i, y_i) at time $t+i$, $0 \le i \le l$. Note that, the target can move in a cycle or stay at the same location for multiple time instants.

Given the routes of the mobile nodes, the time instant t uniquely specifies the locations of the mobile nodes. If the target is at location (x, y) at time t (i.e., characterized by vertex (x, y, t)), it corresponds to certain energy measurements at the sensor nodes. Let $d(x, y, t)$ denote the probability of detecting a target at location (x, y) at time instant t using the value fusion algorithm on the energy measurements from all sensor nodes. For a given target traversal

$$P \equiv (x_0, y_0, t), \ldots, (x_l, y_l, t+l),$$

the probability of detecting the target is

$$D(P) = 1 - \prod_{i=0}^{l} (1 - d(x_i, y_i, t+i)). \tag{4}$$

Note that, for a certain time instant t, $d(x, y, t)$ can be evaluated by Equation (2).

Due to noise, the sensor network may incorrectly decide that a target is present when there is actually no target in the region. The probability of these incidents are called false alarm probability. Similar with the derivation of Equation(4), the false alarm probability is given by

$$D_f = 1 - \left[Prob \left(\sum_{i=1}^{n} N_i^2 < \eta \right) \right]^T,$$

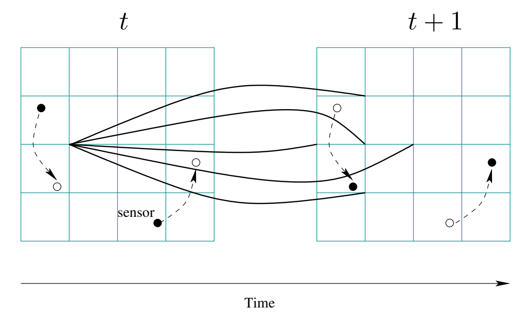


Fig. 3. Paths for time t to $t+1$.

where N_i^2 is the noise energy at node i and T is the least required time the target staying in the region. False alarm probability is dependent of the number of nodes but is independent of the node positions and obstacles. Given a false alarm probability, one can determine the corresponding threshold η. Particularly, detection probability and false alarm probability are closely related. A higher detection probability always comes with a higher false alarm probability.

Exposure is defined as the least probability of detecting a target over all possible target maneuvers for at least T time instants. Formally, exposure can be defined as follows.

Definition 2. *Exposure*: Let $\mathcal{P}$ be the set of all possible target traversals with $l \ge T$. The exposure of the given deployment (i.e., given routes for the mobile nodes) is

$$\mathcal{E} = \min_{P \in \mathcal{P}} D(P).$$

The corresponding path taken by the target is called exposure path.

To compute the exposure, we map a target traversal as a directed path in the following expansion graph. Let L be the least common multiple of the number of sensing positions in each route. The graph has vertices of the form (x, y, t), $0 \le x \le N-1$, $0 \le y \le N-1$, and $t = 0, 1, \ldots$. For each vertex (x, y, t), there is an outgoing edge to vertices $(x, y, t+1)$, $(x+1, y, t+1)$, $(x-1, y, t+1)$, $(x, y-1, t+1)$, and $(x, y+1, t+1)$ subject to the following two conditions: (i) the operation $t+1$ is performed modulo L, and (ii) if $x+1$, $x-1$, $y+1$, and $y-1$ result in values less than 0 or greater than $N-1$, then the corresponding edges are not added. Each incoming edge to vertex (x, y, t) is assigned a weight equal to $|log(1 - d(x, y, t))|$, where $d(x, y, t)$ is the probability of detecting a target at location (x, y) given the mobile node locations at time t. Note that $1 - d(x, y, t)$ lies between 0 and 1, and thus $log(1 - d(x, y, t))$ is negative.

Fig. 3 shows an example of two time frames for the expansion graph. For clarity, only one set of the edges on a grid point is shown in the figure. In fact, every grid point has five edges connecting to the next time frame. Intuitively, the expansion graph adds a time axis for the deployment. Each time frame of the expansion graph is associated with a snapshot of mobile nodes' positions at the time instant and the edges connecting adjacent time frames denote the five possible moving directions of the target (i.e., moving up, down, right, left, and idling). Hence, target energy detected by mobile nodes at each time instant can be evaluated using the expansion graph and thus the weight on each edge can be determined.

The expansion graph can be expanded for infinite time frames. Obviously, a target traversal P with l time instants has total weight

$$w = \sum_{i=0}^{l} |log(1 - d(x_i, y_i, t + i))|. \tag{5}$$

From definition 2 and Equation (4), finding exposure is to find the least detection probability for a target traversal, which is equivalent to finding the minimum of Equation (5), with $l \geq T$. One can show that the exposure path is the least weight target traversal in the expansion graph with $l \geq T$. Note that the target traversal is not necessary to start from the first time frame of the expansion graph. Once the exposure path is found, by Equation (4), the exposure is

$$\mathcal{E} = 1 - 10^{-w}. \tag{6}$$

Further, the exposure path on expansion graph can be converted to determine target positions versus time. Indeed, the number of time steps in the exposure path can be very large comparing to the required time T and there is no principle indicating how long we should expand the time in the expansion graph. However, we can compute the upper and lower bounds on exposure fairly efficiently. The algorithms to find the upper and lower bounds is presented in the rest of the section.

It is worth to note that obstacles are allowed to be in the region. In the presence of obstacles, the edges in the expansion graph that intersect with obstacles are deleted, and the distortion factor in Equation (1) is applied such that the impact of obstacles is included in energy measurements of the nodes.

Procedure LOWER_BOUND
g : an expansion graph;
p : a path;

for $t = 0$ to $L - 1$
  g = construct_prefix(t);
  p_{1t} = shortest_path(g);
end for
$p_1 = \min\{p_{1t} | 0 \leq t \leq L - 1\}$;
L_1^* = weight(p_1);

for $t = 0$ to $L - 1$
  g = construct_suffix(t);
  p_{2t} = shortest_path(g);
end for
$p_2 = \min\{p_{2t} | 0 \leq t \leq L - 1\}$;
L_2^* = weight(p_2);

$w_l = \max(L_1^*, L_2^*)$;
$lower\_bound = 1 - 10^{-w_l}$;
End Procedure

Function Construct_Prefix(t)
s : a dummy starting point
d : a dummy ending point

1) Generate expansion graph g from time t to $t + T$;
2) Connect s to $B(t)$ with weight zero;
3) Connect $G(t + T)$ to d with weight zero;

return(g);
End Function

Function Construct_Suffix(t)
s : a dummy starting point
d : a dummy ending point

1) Generate expansion graph g from time t to $t + T$;
2) Connect s to $G(t)$ with weight zero;
3) Connect $B(t + T)$ to d with weight zero;

return(g);
End Function

Fig. 4. The procedure for computing lower bound

B. Upper and lower bounds of exposure

Lower bound: Let $B(k)$ denote the set of all the "boundary" grid points of the kth time frame, i.e., the set of vertices in the expansion graph of the form $(0, y, k)$ and $(N - 1, y, k)$, with $0 \leq y \leq N - 1$ and $(x, 0, k)$ and $(x, N - 1, k)$ with $0 \leq x \leq N - 1$. Further, let $G(k)$ denote the set of all the grid points of the kth time frame, i.e., the set of vertices in the expansion graph of the form (x, y, k), $0 \leq x \leq N - 1$, $0 \leq y \leq N - 1$.

Definition 3: A *prefix traversal* of length T starting at time t is a sequence of vertices of the form (x_0, y_0, t), $(x_1, y_1, t+1)$, ... $(x_T, y_T, t + T)$ such that

1) $(x_0, y_0, t) \in B(t)$,
2) $(x_T, y_T, t + T) \in G(t + T)$,
3) for $0 \leq i \leq T$, $0 \leq x_i, y_i \leq N - 1$, and
4) for $0 \leq i \leq T - 1$, (x_{i+1}, y_{i+1}) is either (x_i, y_i) or $(x_i + 1, y_i)$ or $(x_i - 1, y_i)$ or $(x_i, y_i - 1)$ or $(x_i, y_i + 1)$.

Let $\mathcal{P}_{1t}$ be the set of all prefix traversals of length T starting at time t. Let $\mathcal{P}_1 = \bigcup_{t=0}^{L-1} \mathcal{P}_{1t}$ and L_1^* be the weight of the least weight prefix traversal in $\mathcal{P}_1$.

Claim 1: L_1^* is a lower bound of weight on the exposure path.

Proof: Let P^* be a true exposure path with exposure $\mathcal{E}$. Since L is the least common multiple of the number of sensing locations of the mobile nodes, the sequence of locations of all mobile nodes is periodic with period L. Therefore, it is sufficient to consider cases where the target enters the region in the time frames 0 to $L - 1$. One can easily show that the first T steps of P^* must belong to $\mathcal{P}_1$. The claim follows since the weight of P^* must be greater than the weight of the corresponding prefix traversal in $\mathcal{P}_1$. ■

Definition 4: A *suffix traversal* of length T starting at time t is a sequence of vertices of the form (x_0, y_0, t), $(x_1, y_1, t+1)$, ... $(x_T, y_T, t + T)$ such that

1) $(x_0, y_0, t) \in G(t)$,
2) $(x_T, y_T, t + T) \in B(t + T)$,
3) for $0 \leq i \leq T$, $0 \leq x_i, y_i \leq N - 1$, and
4) for $0 \leq i \leq T - 1$, (x_{i+1}, y_{i+1}) is either (x_i, y_i) or $(x_i + 1, y_i)$ or $(x_i - 1, y_i)$ or $(x_i, y_i - 1)$ or $(x_i, y_i + 1)$.

Let $\mathcal{P}_{2t}$ be the set of all suffix traversals of length T starting at time t. Let $\mathcal{P}_2 = \bigcup_{t=0}^{L-1} \mathcal{P}_{2t}$. Let L_2^* be the weight of the least weight suffix traversal in $\mathcal{P}_2$.

Claim 2: L_2^* is a lower bound of weight on the exposure path.

Proof: Let P^* be a true exposure path with exposure $\mathcal{E}$. Since L is the least common multiple of the number of sensing locations of the mobile nodes, the sequence of locations of

Procedure UPPER_BOUND
g : an expansion graph;
p : a path;

for $t = 0$ to $L - 1$
g = construct_graph(t);
p_t = shortest_path(g);
end for
$p_u = \min\{p_t | 0 \leq t \leq L-1\}$;
U^* = weight(p_{U^*});
$upper\_bound = 1 - 10^{-U^*}$;
End Procedure

Function Construct_Graph(t)
s : a dummy starting point
d : a dummy ending point

1) Generate expansion graph g from time t to $t + T + M$;
2) Connect s to $B(t)$ with weight zero;
3) Connect $B(t+T), ..., B(t+T+M)$ to d with weight zero;

return(g);
End Function

Fig. 5. The procedure for computing upper bound

all mobile nodes is periodic with period L. Therefore, it is sufficient to consider cases where the target exits the region in the time frames T to $T + L - 1$. One can easily show that the final T steps of P^* must belong to $\mathcal{P}_2$. The claim follows since the weight of P^* must be greater than the weight of the corresponding suffix traversal in $\mathcal{P}_2$. ■

Note that, L_1^* and L_2^* can be easily determined using Dijkstra's shortest path algorithm. Applying Equation (6), a lower bound on the exposure can then be determined using the weight w_l as follows.

$$w_l = \max(L_1^*, L_2^*).$$

A procedure to compute the lower bound is listed in Fig. 4. To compute L_1^*, an expansion graph is constructed. Consider a target entering the region at time t, for $0 \leq t \leq L-1$. A dummy starting point s is created and connected to $B(t)$, i.e., all the boundary grid points on the t_{th} time frame with weight zero. A dummy end point d is also created and all the grid points in $G(t+T)$, i.e., all the grid points on the $(t+T)th$ time frame, are connected to d with weight zero. Find the shortest path p_{1t} from s to d. Then L_1^* is the weight of the overall shortest path of p_{1t} with $0 \leq t \leq L-1$. Analogously, L_2^* can be determined on the expansion graph that s is connected to the grid points in $G(t)$ and the grid points in $B(t+T)$ are connected to d with zero weight edges. Then, the lower bound can be determined as the larger one of L_1^* and L_2^*.

Upper bound: Assume that we are allowed to expand $T+M+L$ time frames in the expansion graph for a certain integer M due to storage or other constraints. Let $\mathcal{P}_t$ be the set of all target traversals with length l starting at time t and $T \leq l \leq T+M$. Let $\mathcal{P}_u = \bigcup_{t=0}^{L-1} \mathcal{P}_t$ and U^* be the weight of the least weight target traversal in $\mathcal{P}_u$.

Claim 3: U^* is an upper bound of weight on the exposure path.

Proof: Let P^* denote the exposure path. From the definition, P^* is the target traversal with length $l \geq T$ as well as the least detection probability. Hence, the weight of P^* must be equal to or less than U^*. ■

It is easy to compute U^* using Dijkstra's shortest path algorithm. A procedure to compute the upper bound on exposure is listed in Fig 5. The procedure is similar with that for computing lower bound except the constructed expansion graph. Since a eligible target traversal can have length l with $T \leq l \leq T+M$, a expansion graph is constructed as follows. Consider a target enters the region at time t. A dummy starting point s and a dummy ending point d are created. Connect s to the boundary grid points in $B(t)$ and connect all the boundary grid points in $B(t+T), ..., B(t+T+M)$ to d. Let p_t be the shortest path on this expansion graph. The upper bound U^* is the weight on the overall shortest path of p_t with $0 \leq t \leq L-1$. A tighter upper bound may be obtained with a larger M. Note that these algorithms are effective for any route and sensing schedule for mobile nodes with and without the presence of obstacles in the region.

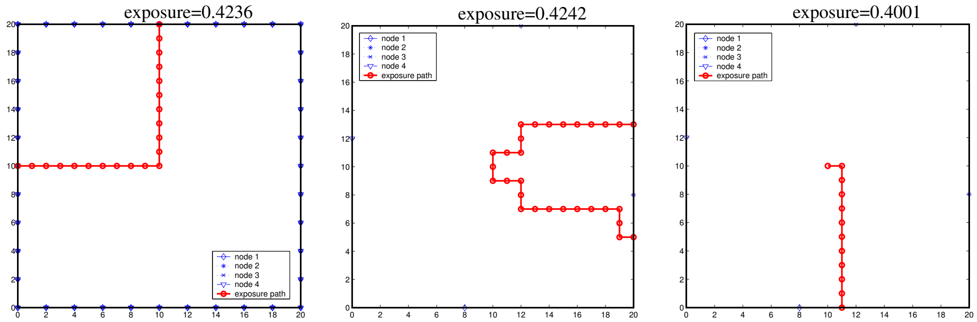
V. Simulations

In the following simulations, we demonstrate the approach to calculate the upper and lower bounds on the exposure in mobile sensor networks. We use four mobile nodes and the routes of the mobile nodes are given in a 20×20 region. The target energy is assumed to be $K = 150$ with decay factor $k = 2$, and the noise is assumed to be AWGN with mean zero and variance one. The threshold η for fusion is chosen such that the false alarm probability is 5%. In addition, we assume that the target must stay in the region for at least 100 time units to accomplish its task.

A. No Obstacle

In the first simulation, the mobile nodes are set up to mimic border patrol for a region without the presence of obstacles. We assume that the nodes can traverse two grid edges and the target can either traverse one grid edge or idling during one time instant. Although mobile nodes' speed is twice faster than the target, the speed of mobile nodes would not affect our exposure evaluation approach. Mobile nodes move counter-clockwise starting from the four corners of the region at time zero. The paths of the upper bounds with $M = 0$ and $M = 120$ and the lower bound are illustrated in Fig. 6, and the corresponding target positions versus time are given in Table I. For clarity, we only list the time and coordinates when the target changes activities. From the figures, the upper bound with $M = 120$ and $M = 0$ are 0.4236 and 0.4242, respectively and the lower bound is 0.4. In this example, the upper bound can be improved with larger M. With $M = 120$, the target enters the region at time 8 and stays in the center of the region until it moves out. Since exposure is the least probability of detecting the target, the target would stay away from the nodes as far as possible on the exposure path. Notice that the target spends 105 time units in the region instead of exactly 100 time units. This is because it wants to choose an appropriate time to leave such that the probability of being detected is low.

In the second simulation, the nodes are arranged to patrol the whole region as illustrated in Fig. 7. The nodes follow



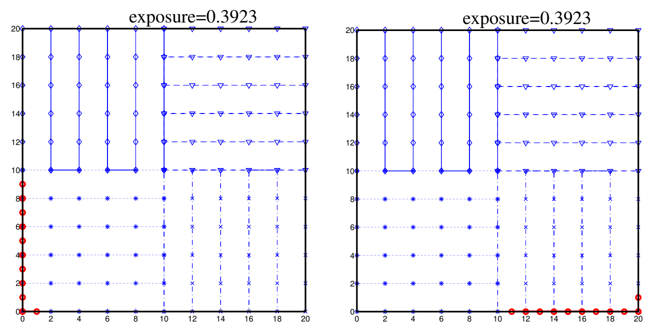
(a) Upper bound with $M = 120$ (b) Upper bound with $M = 0$ (c) lower bound

Fig. 6. Exposure for patrolling boundary.

TABLE I. TRACE OF THE PATHS IN FIG. 6

Upper bound		Upper bound $M = 0$				Lower bound	
t	(x,y)	t	(x,y)	t	(x,y)	t	(x,y)
8	(10,20)	5	(20,5)	20	(10,10)	4	(10,10)
↓	↓	6	(19,5)	↓	...	↓	...
18	(10,10)	7	(19,6)	92	(10,11)	93	(11,10)
↓	...	8	(19,7)	93	(11,11)	↓	↓
103	(9,10)	↓	↓	94	(12,11)	103	(11,0)
↓	↓	15	(12,7)	95	(12,12)		
112	(0,10)	16	(12,8)	96	(12,13)		
		17	(12,9)	↓	↓		
		18	(11,9)	104	(20,13)		
		19	(10,9)				

Notation: ↓ : Traversing, ... : Idling



(a) Upper bound with $M = 0$ (b) Lower bound

Fig. 7. Exposure for patrolling whole region.

TABLE II. TRACE OF THE EXPOSURE PATH IN FIG. 7

t	(x,y)	t	(x,y)	t	(x,y)
55	(0,0)	99	(0,9)	120	(0,0)
↓	...	↓	↓	121	(0,0)
88	(1,0)	105	(0,3)	122	(1,0)
89	(0,0)	↓	↓	123	(0,0)
90	(0,0)	111	(0,9)	↓	...
↓	↓	↓	↓	154	(0,0)

Notation: ↓ : Traversing, ... : Idling

the routes in the sense of zigzag from the center of the region to the boundary and then go back to the center in the inverse direction. Measurements are taken in both forward and backward directions. The results show that both the upper bound with $M = 0$ and the lower bound of the exposure are 0.392, Thus, we have the true exposure path for a target staying in the region at least 100 time units. There is no need to examine the upper bound with $M > 0$, since the worst case exposure path has been found. The trace of the positions versus time for the exposure path is shown in Table II.

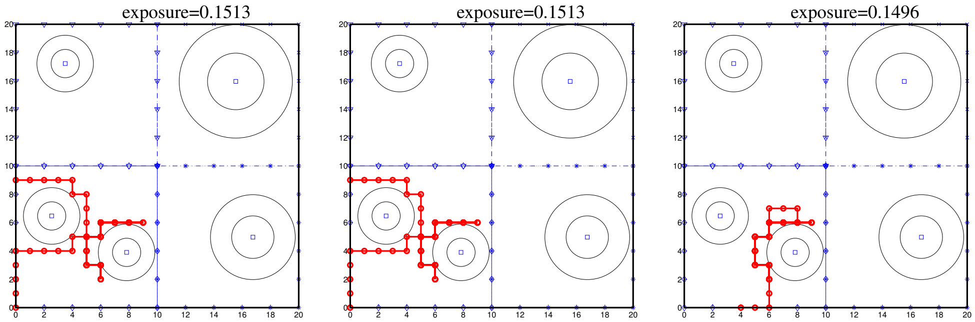
From the simulations, the exposure of the boundary patrol is higher than that of the whole region patrol. However, the exposure path of the boundary patrol allows the target to go into the interior of the region while the exposure path of the whole region patrol is only on the boundary. Choosing the optimal routes for mobile nodes is an challenging problem. In certain cases, if the target is not hostile, one may favor the border patrol to increase the probability of detecting the presence of the target. In other cases, one may choose the whole region patrol to well cover the interior of the region. Although the above simulations only give two typical route plans, our exposure evaluation method is applicable to any number of mobile nodes with various route and sensing schedules.

B. With obstacle

In the presence of obstacles, we place 5 obstacles randomly in the region as shown in Fig. 8. The radii of the obstacles are 2, 3, and 4 with respect to the different sizes of the obstacles in the figure. The radius of the inner circle associated with the energy distortion effect is set to be half of the obstacle radius. Four nodes are arranged to patrol the region counterclockwise from the center of the region. The upper bound with $M = 120$ and $M = 0$ and the lower bound are illustrated in Fig. 8. In this case, the two upper bound paths are identical. Since the lower bound and the upper bound are very close, the upper bound target traversal is possibly very close to the exposure path. For space constraint, we only list the upper bound path versus time in Table III. An interesting observation is that the target traversal takes the shelter between two obstacles, and if the target has to stay for a long time in the region, it repeats its motion with a period equal to the period of the patrol of the nodes.

VI. CONCLUSION

Mobile sensor nodes provide more flexibility and agility in surveillance systems for many situations where traditional stationary sensor networks are inadequate to the practical situations. This paper addresses the problem of evaluating



(a) Upper bound with $M = 120$ (b) Upper bound with $M = 0$ (c) Lower bound

Fig. 8. Exposure path with obstacles.

TABLE III. TRACE OF THE EXPOSURE PATH IN FIG. 8 (A)

t	(x,y)	t	(x,y)	t	(x,y)	t	(x,y)	t	(x,y)
14	(0,0)	↓	↓	42	(6,3)	54	Repeat	99	(6,3)
15	...	31	(9,6)	43	(5,3)	↓	t=34 to	100	(6,2)
16	(0,1)	↓	↓	44	(5,4)	73	t=53	101	(6,3)
↓	↓	34	(6,6)	45	(5,5)	74	Repeat	102	(5,3)
20	(0,4)	35	(6,5)	46	(6,5)	↓	t=34 to	↓	↓
↓	↓	36	(5,5)	47	(6,6)	93	t=53	107	(5,8)
24	(4,4)	37	(5,4)	↓	↓	94	(6,6)	108	(4,8)
25	(4,5)	38	(5,3)	50	(9,6)	95	(6,5)	109	(4,9)
26	(5,5)	39	(6,3)	51	...	96	(5,5)	↓	↓
27	(6,5)	40	(6,2)	52	(8,6)	97	(5,4)	113	(0,9)
28	(6,6)	41	...	53	(7,6)	98	(5,3)		

Notation: ↓ : Traversing, ... : Idling

exposure for a target appearing in a region monitored by a mobile sensor network. As calculating the exposure can be computationally expensive, algorithms using time expansion graph are developed to find the upper and lower bounds on exposure. The experimental results indicate that the algorithms can determine the upper and lower bounds on exposure for any sensor route plan and sensing schedule with and without the presence of obstacles. Since exposure has been used as a metric of the quality of sensor deployments, the algorithms are important for both quality evaluation of mobile sensor networks and route planning for mobile sensor nodes. Our future work is going to plan better routes and sensing positions for a group of mobile nodes such that the exposure can achieve a required threshold. This problem is critical to efficiently use mobile nodes and increase the performance of mobile sensor networks.

REFERENCES

[1] V. Phipatanasuphorn and P. Ramanathan, "Vulnerability of sensor networks to unauthorized traversal and monitoring," *IEEE Transactions on Computers*, vol. 53, no. 3, pp. 364–369, Mar. 2004.

[2] T. Clouqueur, K. K. Saluja, and P. Ramanathan, "Fault-tolerance in collaborative sensor networks for target detection," *IEEE Transactions on Computers*, vol. 53, no. 3, pp. 320–333, Mar. 2004.

[3] A. Mainwaring, J. Polastre, R. Szewczyk, D. Culler, and J. Anderson, "Wireless sensor network for habitat monitoring," in *Proceedings of ACM International Conference of Wireless Sensor Networks and Applications*, 2002.

[4] P. Juang, et al, "Energy-efficient computing for wildlife tracking: Design tradeoffs and early experiences with Zebranet," in *Proceedings of ASPLOS*, 2002.

[5] P. Santi and D. M. Blough, "The critical transmitting range for connectivity in sparse wireless ad hoc networks," *IEEE Transactions on Mobile Computing*, vol. 2, Jan. 2003.

[6] S. Meguerdichian, F. Koushanfar, G. Qu, and M. Potkonjak, "Exposure in wireless ad-hoc sensor networks," in *Proc. of MOBICOM*, July 2001, pp. 139–150.

[7] T. Clouqueur, V. Phipatanasuphorn, P. Ramanathan, and K. K. Saluja, "Sensor deployment strategy for detection of targets traversing a region," *Journal on Mobile Networks and Applications*, pp. 453–461, 2003.

[8] G. Veltri, Q. Huang, G. Qu, and M. Potkonjak, "Minimal and maximal exposure path algorithms for wireless embedded sensor networks," in *Proceedings of the first international conference on Embedded networked sensor systems*, Los Angeles, CA, Nov. 2003.

[9] T. Clouqueur, P. Ramanathan, and K. K. Saluja, "Exposure of variable speed targets through a sensor field," in *Proc. of the 6th Ann. Conf. on Information Fusion*, Aug. 2003.

[10] K. Chakrabarty, S. S. Iyengar, H. Qi, and E. Cho, "Grid coverage for surveillance and target location in distributed sensor networks," *IEEE Transactions on Computers*, vol. 51, Dec. 2002.

[11] Y. Zou and K. Chakrabarty, "A practical sensor deployment strategy based on virtual forces," in *Proceedings of INFOCOM*, vol. 2, Apr. 2003.

[12] S. S. Dhillon and K. Chakrabarty, "Sensor placement for effective coverage and surveillance in distributed sensor networks," in *Proc. IEEE Wireless Communications and Networking Conference*, 2003, pp. 1609–1614.

[13] R. Shah, S. Roy, S. Jain, and W. Brunette, "Data MULEs: modeling a three-tier architecture for sparse sensor networks," in *Proc. of the first IEEE International Workshop on Sensor Network Protocols and Applications*, May 2003, pp. 30–41.

[14] D. K. Goldenberg, J. Lin, A. S. Morse, B. E. Rosen, and Y. R. Yang, "Towards mobility as a network control primitive," in *ACM International Symposium on Mobile Ad Hoc Networking and Computing*, Roppongi, Japan, May 2004, pp. 163–174.

[15] W. Zhao, M. Ammar, and E. Zegura, "A message ferrying approach for data delivery in sparse mobile ad hoc networks," in *ACM International Symposium on Mobile Ad Hoc Networking and Computing*, Roppongi, Japan, May 2004, pp. 187–198.

[16] A. Kansal, A. A. Somasundara, D. D. Jea, M. B. Srivastava, and D. Estrin, "Intelligent fluid infrastructure for embedded networks," in *MobiSys*, Boston, MA, June 2004, pp. 111–124.

[17] G. T. Sibley, M. H. Rahimi, and G. S. Sukhatme, "Robomote: A tiny mobile robot platform for large-scale ad-hoc sensor networks," in *Proceedings of the 2002 IEEE International Conference on Robotics and Automation*, Washington, DC, May 2002, pp. 1143–1148.

[18] M. Rahimi, H. Shah, G. S. Sukhatme, J. Heidemann, and D. Estrin, "Studying the feasibility of energy harvesting in a mobile sensor network," in *Proceedings of IEEE International Conference on Robotics and Automation*, 2003.

[19] T. D. Parsons, "Pursuit-evasion in a graph," in *Theory and Application of Graphs*, Y. Alani and D. R. Lick, Eds. Springer-Verlag, 1976, pp. 426–441.

[20] J. P. Hespanha, H. J. Kim, and S. Sastry, "Multiple-agent probabilistic pursuit-evasion games," in *Proc. of the 38th Conf. on Decision and Control.*, Dec. 1999.

[21] M. Hata, "Empirical formula for propagation loss in land mobile radio service," *IEEE Transactions on Vehicular Technology*, vol. 29, pp. 317–325, Aug. 1980.