Adaptive Power and Rate Allocation for Service Curve Assurance in DS-CDMA Network

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***Abstract*– The paper describes schemes for forward and reverse links in a Direct Sequence Code Division Multiple Access (DS-CDMA) based cellular network. The primary objective is to meet the diverse quality of service (QoS) needs of mobile hosts (MHs) and the secondary objective is to maximize the system throughput. The QoS needs of the MHs are modelled using the notion of a service curve. Furthermore, a notion of deviation is introduced as a measure of meeting service curve.**

The scheme proposed in this paper jointly adapts the transmitted power and the number of spreading codes assigned to each MH for receiving/transmitting its data bits. The scheme imposes practical constraints including bounds on the transmitted power for BS and MHs, a bound on the number of spreading codes that a MH can handle, and minimum Signal to Interference plus Noise Ratio (SINR) at the receiver. The proposed solutions are evaluated using discrete event simulations. The simulation results characterize the performance of the proposed solutions for several instances of the practical constraints.

Key words: Wireless networks, Multimedia systems, Quality of service differentiation, Power Control, Multi-rate Networks, 3G systems, Forward link, Reverse link.

1 Introduction

Next generation wireless and wireline network must concurrently support applications with diverse quality of service (QoS) requirements. For example, the QoS needs of data-oriented

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applications such as telnet, ftp, and email are considerably different from those of streaming applications such as Voice-over-IP and real audio. The need to support such diverse QoS requirements has been recognized in wireline network research for well over a decade and numerous solutions have been proposed to provide such a support. A QoS-aware packet scheduling policy based on Packet Fair Queueing (PFQ) is a key component of these solutions [1][2][3][4].

In wireless networks, on the other hand, research on diverse QoS support is fairly recent. For instance, Lu, Bharghavan, and Srikanth [5] were one of the first to propose a fair scheduling policy for wireless networks that tries to approximate the Weighted Fair Queueing (WFQ) policy [2]. Following that work, Ng, Stoica, and Zhang in [6], Ramanathan and Agrawal in [7] and Chang and Chen in [1] have also proposed approaches for adapting various wireline PFQ algorithms to wireless networks. As in all the wireline PFQ algorithms, the solutions in [1][2][5][6][7] assume Time Division Multiplex Access (TDMA) scheme. In contrast, we consider a Direct Sequence Code Division Multiple Access (DS-CDMA) based cellular network in this paper.

In a DS-CDMA network, multiple transmissions can occur simultaneously, each using a pre-assigned unique "spreading code". At any time instant, a receiver gets attenuated signals from all the simultaneous transmissions. From these signals, a receiver decodes the information in a particular transmission using the transmission's unique spreading code. In this process, the signals from other transmissions manifest as interference.

QoS differentiation in a DS-CDMA network can be achieved by careful allocation of two resources: *transmitted power* and *rate*. Power allocation affects the relative received strength of signals at the receiver, which in turn determines the bit error rate in decoding the corresponding information. The rate allocation determines the number of bits that can be transmitted per unit time. In a DS-CDMA network, rate allocation can be done in one of two ways: (i) by changing a parameter called the processing gain, and (ii) by changing the number of spreading codes used for a transmission. In this paper, we use the latter approach for rate allocation. This latter approach is commonly referred to as the multi-code scheme.

Prior work in DS-CDMA networks on adaptive power and rate allocation often focus on throughput maximization. For example, in [8][9], Oh, Wasserman, and Olsen, use a game theoretic approach to dynamically allocate power and processing gain to maximize the total reverse link throughput. In [10], Jafar and Goldsmith characterize optimal power and code allocation to maximize the reverse link throughput in a multicode DS-CDMA network. In

[11], Elaoud and Ramanathan describe a scheme to adapt the SINR requirements to meet the delay needs of MHs while exploiting the channel quality. In [12], Tong, Ramanathan, and Sayeed describe a heuristic rate and power adaptation scheme for dealing with MHs with diverse QoS requirements represented in the form of service curve. The scheme in [12] is also for reverse link.

In [13], Song and Mandayam develop a solution that adapts rate and power for forward link. The QoS requirements of MHs are assumed to be mapped onto a family of utility functions that reflect the tradeoff between total system throughput and fairness. The allocation of rate and power is determined as a solution to a constrained optimization problem. Instead of a standard search algorithm, the paper proposes a hierarchical approach for efficiently computing the optimal solution. The nature of the constraints and the problem formulation in [13] is similar to that in this paper. The approach in this paper is, however, different in following ways. First, in this paper, the QoS requirements of MHs are modeled using service curves as opposed to utility functions. As a result, unlike in [13], recent history of the allocation is taken into account while allocating power and rate at a particular time instant. Second, in this paper, we consider both forward and reverse links. Third, our solution approach clearly identifies cases when the constrained optimization problem has no feasible solution and proposes an allocation mechanism even for those cases. The paper in [13] implicitly assumes that there is always a feasible solution to the constrained optimization problem.

In this paper, we model the QoS needs of a MH using the notion of a *service curve* [14]. The service curve specifies the minimum number of bits a MH must transmit/receive in given time interval in order to meet its QoS requirements. Due to fluctuations in the quality of a wireless channel, it is not possible to guarantee that a MH will always be able to transmit/receive the number of bits specified by the service curve. We propose forward and reverse link schemes that dynamically allocate power and rate to minimize the deviation from the service curve requirements of a MH. In situations where the wireless channel has adequate capacity to meet the service curve requirements of all the MHs, the proposed scheme also maximizes the system throughput.

The rest of this paper is organized as follows. In Section 2, we briefly review relevant background material on service curves and DS-CDMA networks. Then, in Section 3, we formally describe the system model. The proposed solution is described in Section 4 and simulation results evaluating our approach are presented in Section 5. The paper concludes with Section 6.

2 Background

2.1 Notions of Service & Deadline Curves

In wireline networks, QoS requirements have often been modeled using the notion of a service curve. Let $S_i(.)$ denote the service curve of an active session i. We say that the QoS requirements of session i are met at time t if its service curve S_i is assured in the following sense.

Definition 1: Let $W_i(u, v)$ denote the amount of service a session i has received in the interval $[u, v)$. Then, its service curve $S_i(.)$ is said to be assured at time t_2, if there exits a $t_1 < t_2$, where t_1 is the start of a backlog period[1] of session i such that $W_i(t_1, t_2) \geq S_i(t_2 - t_1)$.

Algorithmically, given a service curve $S_i(.)$, one can efficiently compute at any time s, a function called a *deadline curve*, $D_i(u)$, which specifies for all time $u > s$, the minimum amount of service session i must receive in interval $[0, u)$ in order to assure the service curve $S_i(.)$ at time u. Formally, if $B_i(s)$ is the set of start of backlog periods of session i prior to time s, then

$$D_i(u) = \min_{v \in B_i(s)} \{S_i(u - v) + W_i(0, v)\}. \tag{1}$$

As described in [15], the following iterative algorithm can be used to maintain $D_i(u)$. At the start of the first backlog period, initialize $D_i(u) = S_i(u)$. Subsequently, at the start of the k^{th} backlog period at time a_i^k update $D_i(u)$ as follows:

$$D_i(a_i^k; u) = \min \left\{ D_i(a_i^{k-1}; u), S_i(u - a_i^k) + W_i(0, a_i^k) \right\}$$

for $u \geq a_i^k$, and where $D_i(a_i^{k-1}; u)$ is deadline curve as updated at time a_i^{k-1}.

2.2 SINR Computation for Matched Filter Receiver

Let C denote the set of spreading codes concurrently being used at any particular time t. Consider the data transmission using the spreading code $s \in C$. Let $P(s)$ denote the power used for this transmission. Let $g(s)$ denote the amount of signal attenuation, i.e.,

[1] In the above definition, a session is said to be backlogged if it has packets to transmit.

path loss, for this transmission. Then, the Signal to Interference plus Noise Ratio (SINR) for Matched Filter decoding of data transmitted using s is

$$\mathrm{SINR}(s) = \frac{(s \cdot s)^2 g(s) P(s)}{\sum_{s' \in C, s' \neq s} (s' \cdot s)^2 g(s') P(s') + (s \cdot s) N_0} \tag{2}$$

where $(s \cdot s')$ denotes the inner product of spreading codes s and s' and N_0 is power spectral density of background AWGN. In this paper, we assume that the spreading codes are non-orthogonal and the processing gain $G = \frac{(s \cdot s)}{(s' \cdot s)}$ for all $s' \neq s$.

3 Problem Formulation

We consider a single cell of a DS-CDMA based cellular network. We assume that there are K active mobile hosts (MHs) in the cell. The QoS need of each MH is modeled using service curve. Let $S_i(t)$ and $D_i(t)$ respectively denote the service curve and deadline curve of MH_i. Also let $W_i(t)$ denote the total amount of service (i.e., transmitted or received) MH_i has received in the interval $(0, t]$.

Define the *deviation from service curve* as

$$V_i(t) = \max\{D_i(t) - W_i(t), 0\} \tag{3}$$

That is, the deviation is zero at time t if MH_i has received more service than $D_i(t)$. Otherwise, the deviation equals the shortfall in the amount of service MH_i has received in $(0, t]$ as compared to $D_i(t)$. Define the *system deviation* as $V(t) = \sum_{i=1}^{K} V_i(t)$.

In our approach, power and rate adaptation occurs once every Δ time units, where Δ is less than or equal to the coherence time of the channel. For reverse link, let $n_i(t)$ and $P_i(t)$ denote the number of spreading codes and transmit power used by MH_i at time t. Similarly, for forward link, let $n_i(t)$ and $P_i(t)$ denote the number of spreading codes and transmit power used by the BS for MH_i at time t. When the time instant under consideration is clear from the context, we respectively use n_i and P_i instead of $n_i(t)$ and $P_i(t)$.

Due to typical practical limitations, we impose the following constraints.

- For forward link, there is a bound P_{max} on the transmit power of the BS, i.e.,$\forall t$: $\sum_i P_i(t) \leq P_{max}$. Similarly, for reverse link, there is bound $P_{i,max}$ on the transmit power of each MH_i, i.e., $\forall t$: $P_i(t) \leq P_{i,max}$.

- Each MH has a limit M_i on the number of spreading codes it can concurrently transmit or receive.

- There is a lower bound γ on the *Signal to Interference plus Noise Ratio* (SINR) that needs to be achieved for successful decoding of each transmitted frame.

In addition, we make the following modelling assumptions.

- The quality of the wireless channel between a MH and the BS fluctuates over time. We assume that these fluctuations can be characterized using a flat Rayleigh fading model.

- All MHs and the BS use Matched Filter (MF) receiver to decode the transmitted frames.

- We assume one frame of data can be transmitted by a MH using one spreading code in Δ time units.

Given these constraints and assumptions, the problem solved at any given time instant t can be succinctly stated as follows.

Maximize $I_{\{V=0\}} \cdot \sum_i n_i(t) - V$

Subject to:

1. Meet the power constraints: That is,
   For reverse link, $\forall i : 0 \leq P_i(t) \leq P_{i,max}$;
   For forward link, $\forall i : 0 \leq P(t) \leq P_{max}$.

2. Meet the rate constraint, i.e. $\forall i$: $n_i \leq M_i$

3. Meet the SINR constraint of each transmission, i.e. If $n_i > 0$, then $\text{SINR}_i \geq \gamma$

4. Deviation equality $V = \sum_{i=1}^{K} \max\{D_i(t+\Delta) - W_i(t+\Delta), 0\}$

$I_{\{V=0\}}$ is an indicator function which is 1 if $V = 0$ and 0 otherwise.

The proposed optimization problem will maximize system throughput when all MHs' service curve requirement can be met, otherwise, it will minimize system deviation.

4 Proposed Solution

As stated above, our solution re-allocates rate and transmit power once every Δ time units. The primary objective of the re-allocation at time t is to minimize the system deviation from service curve at time $t+\Delta$. The secondary objective is to maximize the total throughput.

Let F be the number of bits the BS can transmit using one spreading code in $(t, t+\Delta)$. Then

$$W_i(t+\Delta) = W_i(t) + n_i(t, t+\Delta)F,$$

and the deviation from service curve at $t+\Delta$ is

$$V_i(t+\Delta) = \max\{D_i(t+\Delta) - W_i(t) - n_i(t, t+\Delta)F, 0\}.$$

Therefore, $V_i(t+\Delta) = 0$ if

$$n_i(t, t+\Delta) \geq \max\left\{\left\lceil \frac{D_i(t+\Delta) - W_i(t)}{F} \right\rceil, 0\right\}.$$

Due to the rate constraint, if $\left\lceil \frac{D_i(t+\Delta)-W_i(t)}{F} \right\rceil > M_i$, the system deviation cannot be made 0 at $t+\Delta$. In this case the minimum system deviation is achieved if $n_i(t, t+\Delta) = M_i$.

Let

$$\begin{aligned} m_i(t, t+\Delta) &= \max\left\{\left\lceil \frac{D_i(t+\Delta) - W_i(t)}{F} \right\rceil, 0\right\} \\ l_i(t, t+\Delta) &= \min\{M_i, m_i(t, t+\Delta)\}. \end{aligned}$$

From the above discussion, minimum system deviation is achieved when $n_i(t, t+\Delta) = l_i(t, t+\Delta)$.

In later discussions, we use the following simpler notations:

$$\begin{aligned} m_i &\equiv m_i(t, t+\Delta) \\ l_i &\equiv l_i(t, t+\Delta) \\ n_i &\equiv n_i(t, t+\Delta) \\ \Omega &\equiv \Omega(t, t+\Delta) \\ \mathbf{n} &\equiv (n_1, n_2, \ldots, n_K) \\ \mathbf{P} &\equiv (P_1, P_2, \ldots, P_K) \end{aligned}$$

Lemma 1: An optimal solution (n^*, P^*) of the constrained optimization problem

> **Minimize** $\sum_i \max\{D_i(t+\Delta) - W_i(t+\Delta), 0\}$
> **Subject to** the power, rate and SINR constraints.

is also an optimal solution for the problem

> **Minimize** $\sum_i \max\{(m_i - n_i), 0\}$
> **Subject to** the power, rate and SINR constraints and $0 \le n_i \le l_i$ for all i

Proof: See Appendix A. ■

Theorem 1: The solution to the following constrained optimization problem also minimizes system deviation.

> **Maximize** throughput $\Omega = \sum_i n_i$
> **Subject to** power, rate and SINR constraints and $0 \le n_i \le l_i$ for all i.

Proof:

The proof follows from the lemma 1 and the observation that minimize $\sum_i \max\{m_i - n_i, 0\}$ = maximize throughput $\Omega = \sum_i n_i$.

As m_i is a fixed number in each time slot, to minimize $\sum_i (m_i - n_i)$ is thus equal to maximize throughput $\Omega = \sum_i n_i$. ■

Figure 1 shows a high level description of our solution to meet the primary and secondary objectives. The optimality of this solution follows directly from Theorem 1. In this solution, we first check whether there exists a feasible allocation of codes and power that meets the service curve of all MHs at time $t + \Delta$. If yes, the system deviation at $t + \Delta$ is 0 and thus minimum. Therefore, the solution invokes the throughput phase where we maximize the throughput subject to keeping the system deviation 0. On the other hand, when system deviation cannot be made 0, then the solution invokes the deviation phase where it minimizes the system deviation as characterized in Theorem 1. The algorithm to check feasibility and identify the solution for constrained optimal problems in throughput and deviation phase are different for forward and reverse links. In the following two subsections, we consider forward and reverse link individually and describe the algorithm in detail.

Check feasible for $\mathbf{n} = (l_1, l_2, \ldots, l_K)$
If feasible
*/*throughput phase*/*
Maximize throughput $\Omega = \sum_i n_i$
Subject to: power, rate and SINR constraints and $l_i \leq n_i \leq M_i$
Else
*/*deviation phase*/*
Maximize throughput $\Omega = \sum_i n_i$
Subject to: power, rate and SINR constraints and $0 \leq n_i \leq l_i$
END
Compute $\mathbf{P}$ for $\mathbf{n}$

Figure 1: High level solution.

4.1 Forward Link

Without loss of generality, let $g_1 \geq g_2 \geq \ldots \geq g_K$, and g_i is channel gain of MH_i, $1 \leq i \leq K$.

Lemma 2: In a forward link transmission, a code assignment $\mathbf{n} = (n_1, n_2, \ldots, n_K)$ is feasible, i.e., there exists a feasible power assignment $\mathbf{P} = (P_1, P_2, \ldots, P_K)$ such that rate, power and SINR constraints are satisfied, if and only if

$$\frac{\gamma\sigma^2}{G+\gamma}\sum_{i=1}^{K}\frac{n_i}{g_i} \leq P_{\max}\left(1 - \frac{\gamma}{G+\gamma}\sum_{i=1}^{K} n_i\right) \quad \text{and} \tag{4}$$

$$\frac{\gamma}{G+\gamma}\sum_{i=1}^{K} n_i < 1 \tag{5}$$

where G is the processing gain and $\sigma^2 = GN_0$.

Furthermore, if $\mathbf{n}$ is feasible, one of the optimal solutions of $\mathbf{P}$ is

$$\forall i : P_i = \frac{\gamma\sigma^2}{G+\gamma-\gamma\Omega}\frac{n_i}{g_i} \tag{6}$$

Proof:

For forward link transmission, the SINR constraint of each code is

$$\forall i : \qquad \gamma \leq \frac{g_i G P_i/n_i}{g_i P - g_i P_i/n_i + \sigma^2} = \frac{G P_i/n_i}{P - P_i/n_i + \frac{\sigma^2}{g_i}} \tag{7}$$

where P_i is the total power that used by BS for MH_i and P_T is transmitted power of BS, i.e., $P_T = \sum_i P_i$.

Solving the linear set of inequalities in (7) over $K+1$ variables $(P_1, P_2, \ldots, P_K, P_T)$, we get

$$P_T \geq \left(\frac{\gamma\sigma^2}{G+\gamma}\sum_{i=1}^{K}\frac{n_i}{g_i}\right)\Big/\left(1-\frac{\gamma}{G+\gamma}\sum_{i=1}^{K}n_i\right) \tag{8}$$

Sufficiency:

On right side of (8), the numerator is positive. Because of (5), the denominator is also positive. Therefore $P_T \geq 0$.

By setting

$$P_T = \left(\frac{\gamma\sigma^2}{G+\gamma}\sum_{i=1}^{K}\frac{n_i}{g_i}\right)\Big/\left(1-\frac{\gamma}{G+\gamma}\sum_{i=1}^{K}n_i\right) \tag{9}$$

$P_T \leq P_{max}$ from (4). Therefore $(n_1, n_2, \ldots, n_K)$ is feasible.

Necessary:

Combining power constraint and (8),

$$P_{\max} \geq P_T \geq \left(\frac{\gamma\sigma^2}{G+\gamma}\sum_{i=1}^{K}\frac{n_i}{g_i}\right)\Big/\left(1-\frac{\gamma}{G+\gamma}\sum_{i=1}^{K}n_i\right) \tag{10}$$

That is, (4) holds.

Again, from power constraint, $0 \leq P_T \leq P_{max}$, P_T must be a non-negative number. On right side of inequality (8), the numerator is positive. Therefore, the denominator should also be positive, i.e., (5) holds. That is, if $(n_1, n_2, \ldots, n_K)$ is feasible then (4) and (5) are satisfied. ■

Lemma 3: If $(n_1, \ldots, n_j+1, \ldots, n_K)$ is feasible, then $(n_1, \ldots, n_i+1, \ldots, n_K)$ is also feasible for all $i < j$ such that $n_i + 1 \leq M_i$

Proof:

Since $(n_1, n_2, \ldots, n_K)$ is feasible, from (10)

$$P_{\max} \geq \frac{\gamma\sigma^2}{G+\gamma-\gamma\Lambda}\left(\sum_i \frac{n_i}{g_i} + \frac{1}{g_j}\right) \tag{11}$$

Sort MHs according to their channel condition such that $g_1 \geq g_2 \geq \ldots \geq g_K$
Check feasible for $\mathbf{n} = (l_1, l_2, \ldots, l_K)$
If feasible
  */*Invoke throughput phase*/*
  Find maximum j where $j \leq K$ such that
    $(M_1, M_2, \ldots, M_{j-1}, l_j, l_{j+1}, \ldots, l_K)$ is feasible and
    $(M_1, M_2, \ldots, M_{j-1}, M_j, l_{j+1}, \ldots, l_K)$ is not feasible.
  Find maximum n_j where $n_j \leq M_j$ such that
    $(M_1, M_2, \ldots, M_{j-1}, n_j, l_{j+1}, \ldots, l_K)$ is feasible.
    $(M_1, M_2, \ldots, M_{j-1}, n_j + 1, l_{j+1}, \ldots, l_K)$ is not feasible.
  $\mathbf{n} = (M_1, M_2, \ldots, M_{j-1}, n_j, l_{j+1}, \ldots, l_K)$
Else
  */*Invoke deviation phase*/*
  Find maximum j where $j \leq K$ such that
    $(l_1, l_2, \ldots, l_{j-1}, 0, \ldots, 0)$ is feasible and
    $(l_1, l_2, \ldots, l_{j-1}, l_j, 0, \ldots, 0)$ is not feasible.
  Find maximum n_j where $0 \leq n_j \leq l_j$ such that
    $(l_1, l_2, \ldots, l_{j-1}, n_j, 0, \ldots, 0)$ is feasible.
    $(l_1, l_2, \ldots, l_{j-1}, n_j + 1, 0, \ldots, 0)$ is not feasible.
  $\mathbf{n} = (l_1, l_2, \ldots, l_{j-1}, n_j, 0, \ldots, 0)$
End If
Compute $\mathbf{P}$ for $\mathbf{n}$

Figure 2: Solution of forward link.

where $\Lambda = \sum_i n_i + 1$.

Since $g_i \geq g_j$, (11) implies

$$P_{\max} \geq \frac{\gamma\sigma^2}{G + \gamma - \gamma\Lambda}(\sum_i \frac{n_i}{g_i} + \frac{1}{g_i}) \tag{12}$$

■

Lemma 3 implies that, it is not worse to give as many codes as possible to MHs with better channel.

Figure 2 shows the solution scheme for forward link. Throughput phase is invoked if code assignment $\mathbf{l} = (l_1, l_2, \ldots, l_K)$ is feasible. In this case, since $\forall i : n_i \geq li$, every MH_i must get at least l_i codes. The optimal solution is $\mathbf{n}^* \geq \mathbf{l}$.

According to lemma 3, in an optimal solution, MH_i can be given as much codes as possible before MH_{i+1} can get an additional code beyond l_{i+1}. The limit on number of codes that can be assigned to MH_i is M_i. Thus the solution to the constrained optimization problem in throughput phase can be obtained by finding maximum j and maximum n_j, where $j \leq K$ and $l_j \leq n_j \leq M_j$, such that code assignment $(M_1, M_2, \ldots, M_{j-1}, n_j, l_{j+1}, \ldots l_K)$ is feasible.

Similarly, deviation phase is executed when code assignment $\mathbf{l} = (l_1, l_2, \ldots, l_K)$ is not feasible. This means that the optimal solution is $\mathbf{n}^* < \mathbf{l}$. The solution to the constrained optimization problem in deviation phase can be obtained by finding maximum j and maximum n_j, where $j \leq K$ and $0 \leq n_j \leq l_j$, such that code assignment $(l_1, l_2, \ldots, l_{j-1}, n_j, 0, \ldots, 0)$ is feasible.

Any search algorithm can be used to find j and n_j in throughput and deviation phase. For example, in deviation phase, first, using binary searching algorithm to find j such that $(l_1, \ldots, l_{j-1}, 0, \ldots, 0)$ is feasible and $(l_1, \ldots, l_{j-1}, l_j, 0, \ldots, 0)$ is not feasible. This can be done in $\log K$ steps. Then using binary search algorithm to find n_j such that $(l_1, \ldots, l_{j-1}, n_j, 0, \ldots, 0)$ is feasible and $(l_1, \ldots, l_{j-1}, n_j + 1, 0, \ldots, 0)$ is not feasible. This can be done in $\log M_i$ steps. Since in each step, (4) and (5) are checked, its computational complexity is $O(K)$. Let $M = \max_i M_i$. Thus one can find the solution in $O(\log K + \log M)$.

As mentioned earlier, the bound M_i on the number of spreading codes MH_i can handle arises due to power and computational constraints. As technology improves, this bound tends to increase. The basic structure of the above solution does not change even in the limit as M_i tends to infinity for all i. Specifically, as $M_i \to \infty$, $l_i = m_i$ for all i. Therefore, in the first step, the algorithm checks whether $(m_1, \ldots, m_K)$ is feasible. If this is not feasible, then the algorithm enters the deviation phase where the solution is the same as before. On the other hand, if $(m_1, \ldots, m_K)$ is feasible, them the algorithm enters throughput phase. In this phase, the algorithm will find maximum n_1 such that $m_1 \leq n_1$ and $(n_1, m_2, \ldots, m_K)$ is feasible. That is, the additional spreading codes beyond meeting all service curves will be assigned to the MH with the best channel.

4.2 Reverse Link

Assume without loss of optimality that all codes used by a MH have the same transmit power. Then the SINR constraint is

$$\forall i \text{ and } n_i \neq 0: \quad \gamma \leq \frac{G g_i P_i / n_i}{\sum_{j \neq i} g_j P_j + \frac{n_i - 1}{n_i} g_i P_i + \sigma^2} \tag{13}$$

Let $r_i = g_i P_i$ be the received power of MH_i. Then the SINR constraint becomes

$$\forall i \text{ and } n_i \neq 0: \quad \gamma \leq \frac{G r_i / n_i}{\sum_{j \neq i} r_j + \frac{n_i - 1}{n_i} r_i + \sigma^2} \tag{14}$$

Observe that, on the receiver side (BS), only received power need to be considered.

Further, note that, (14) can be simplified to

$$\frac{r_i}{n_i} \geq \frac{\gamma \sigma^2}{G + \gamma - \gamma \Omega} \tag{15}$$

Define

$$R_C(\Omega) = \frac{\gamma \sigma^2}{G + \gamma - \gamma \Omega} \tag{16}$$

Lemma 4: For reverse link transmission, a code assignment $\mathbf{n} = (n_1, n_2, \ldots, n_K)$ is feasible if and only if

$$\forall i: \qquad \frac{n_i \gamma \sigma^2}{G + \gamma - \gamma \sum_i n_i} \leq g_i P_{i,\max} \quad \text{and} \tag{17}$$

$$\frac{\gamma}{G + \gamma} \sum_{i=1}^{K} n_i < 1 \tag{18}$$

Furthermore, when the code assignment $\mathbf{n}$ is feasible, the power assignment is

$$\forall i: \quad P_i = \frac{n_i \gamma \sigma^2 / g_i}{G + \gamma - \gamma \Omega} \tag{19}$$

Proof:

Sufficiency: That is, (17) and (18) are sufficient conditions for a feasible power and code assignment.

Let $P_i = \frac{n_i}{g_i} R_C(\Omega) > 0$. From (15), we know that it meets the SINR constraints. From (17),

$$P_i = \frac{n_i \gamma \sigma^2 / g_i}{G + \gamma - \gamma\Omega} \leq P_{i,\max} \tag{20}$$

thus sufficiency holds.

Necessary: That is, (17) and (18) are necessary conditions for a feasible code assignment.

To meet SINR constraint, (15) must hold. Further, to meet power constraint $0 \leq P_i \leq P_{i,\max}$, r_i should be non-negative. Since the numerator in (15) is always non-negative, the denominator should also be positive. i.e., inequality (18) is necessary condition. From (15),

$$\frac{n_i \gamma \sigma^2 / g_i}{G + \gamma - \gamma\Omega} \leq \frac{r_i}{g_i} \leq P_{i,\max}$$

Hence inequality (17) is a necessary condition.

Using equation (16) as an optimal solution when code assignment $\mathbf{n}$ is feasible, the power assignment is shown in equation (19).

■

Lemma 5: To find optimal solution for reverse link in throughput phase, the problem is rewritten as follows:

Maximize Throughput $\Omega = \sum_i n_i$
Subject to constraints:
(i) $\forall i:$ $0 \leq P_i \leq P_{i,\max}$
(ii) if $n_i > 0$: $\text{SINR}_i \geq \gamma$
(iii) $\forall i:$ $l_i \leq n_i \leq M_i$

It is equivalent to find optimal solution to the following constrained optimization problem:

Maximize Throughput $\Omega = \sum_i n_i$
Subject to constraints:
(i) $\Omega \leq \frac{G+\gamma}{\gamma} - \frac{\sigma^2 l_i}{g_i P_{i,\max}}$
(ii) $\sum_i l_i \leq \Omega \leq \sum_i M_i$
(iii) $\Omega \leq \sum_i \min\{M_i, \lfloor \frac{g_i P_{i,\max}}{R_c(\Omega)} \rfloor\}$

Furthermore, one way to get the code assignment is as follows.

Let $n_i = l_i + x_i$ where $0 \leq x_i \leq M_i - l_i$.

$$x_i = \begin{cases} \min\{M_i, \left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega)} \right\rfloor\} - l_i & i < j \\ \Omega - \sum_{k=1}^{K} l_k - \sum_{k=1}^{j-1} x_k & i = j \\ 0 & i > j \end{cases} \tag{21}$$

Where $0 \leq j \leq M_i$ and it is the maximum such that

$$\min\{M_i, \left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega)} \right\rfloor\} - l_i \geq 0 \quad \text{and} \quad \sum_{i=1}^{j} \min\{M_i, \left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega)} \right\rfloor\} \leq \Omega - \sum_{i=1}^{j} l_i$$

Proof: See Appendix B. ■

Lemma 6: The solution to the constrained optimization problem in deviation phase can be obtained by solving the following problem.

Maximize Throughput $\Omega = \sum_i n_i$
Subject to constraints:
(i) $\Omega \leq \frac{G+\gamma}{\gamma}$
(ii) $0 \leq \Omega \leq \sum_i l_i$
(iii) $\Omega \leq \sum_i \min\{l_i, \lfloor \frac{g_i P_{i,\max}}{R_c(\Omega)} \rfloor\}$

Furthermore, one way to get the code assignment is as follows.

Let $n_i = x_i$ where $0 \leq x_i \leq l_i$.

$$x_i = \begin{cases} \min\{l_i, \left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega_2)} \right\rfloor\} & i < j \\ \Omega - \sum_{k=1}^{j-1} x_k & i = j \\ 0 & i > j \end{cases}$$

Where $0 \leq j \leq l_i$ and it is the maximum such that

$$\min\{l_i, \left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega)} \right\rfloor\} \geq 0 \quad \text{and} \quad \sum_{i=1}^{j} \min\{l_i, \left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega)} \right\rfloor\} \leq \Omega$$

Proof:

The proof here is similar to lemma 5, the only difference is the range of n_i because of the condition to invoke deviation phase. In deviation phase, $0 \leq n_i \leq l_i$.

■

Check feasible for $\mathbf{n} = (l_1, l_2, \ldots, l_K)$
If feasible
*/*throughput phase*/*
Use Rev_Determine_n(Ω) to find max feasible Ω
$\sum_i l_i \leq \Omega \leq \min(\sum_i M_i, \frac{G}{\gamma} + 1)$
Else
*/*deviation phase*/*
Use Rev_Determine_n(Ω) to find max feasible Ω
$0 \leq \Omega \leq \min\{\sum_i l_i, \frac{G}{\gamma} + 1\}$
End If
Compute $\mathbf{P}$ for $\mathbf{n}$

Figure 3: Solution of reverse link.

Figure 3 shows an algorithm for optimal power and code allocation in reverse link. According to lemma 5 and 6, the maximum Ω can be found by any search algorithm. Any code assignment that adds up to Ω and satisfies power, rate and SINR constraints is an optimal solution.

Figure 4 shows the algorithm to allocate spreading codes to each MH as mentioned in lemma 5 and 6. Given a value of throughput Ω, the function returns *INFEASIBLE* if there is not a feasible code and power assignment. Otherwise, it returns a possible code assignment $\mathbf{n}$ where for all i, $\text{lower}_i \leq n_i \leq \text{upper}_i$. In the function, $\text{lower}_i = 0$ and $\text{upper}_i = l_i$ for deviation phase and $\text{lower}_i = l_i$ and $\text{upper}_i = M_i$ for throughput phase.

Here again, consider the case where M_i tends to infinity for all i. That is, there is no bound on the number of spreading codes a MH can handle. In this case, $l_i = m_i$ for all i. Therefore, in the first step, the algorithm checks whether $(m_1, \ldots, m_K)$ is feasible. If it is not feasible, then the deviation phase is invoked and the solution is same as before. Otherwise, the throughput phase is invoked. In this phase, although there is no limit on number of codes for each MH, the total number of codes that can be simultaneously used in the system is bounded by the system capacity $\frac{G}{\gamma} + 1$. Thus, in equation (21), $\min\{M_i, \left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega_2)} \right\rfloor\}$ becomes $\left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega_2)} \right\rfloor$. Similarly for algorithm in Figure 3, upper bound for Ω is $\frac{G}{\gamma} + 1$ instead of $\min(\sum_i M_i, \frac{G}{\gamma} + 1)$; for algorithm in Figure 4, upper bound of n_i for throughput phase is $\left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega_2)} \right\rfloor$ instead of $\min\{M_i, \left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega_2)} \right\rfloor\}$.

```
/* Determine n in reverse link according to Ω */
/* Return n if there exist a feasible code and power assignment, */
/* Otherwise, return INFEASIBLE. */
/* Suppose ∀i : n_i = lower_i is a feasible code assignment. */
Rev_Determime_n(Ω)
    Ω* = 0;
    For i = 1 to K
        n_i = lower_i;
        Ω* = Ω* + n_i;
    End For
    For i = 1 to K
        n_i = min{upper_i, ⌊g_i P_{i,max} / R_C(Ω)⌋}
        Ω* = Ω* + (n_i − lower_i);
        If Ω* ≥ Ω;
            n_i = n_i − (Ω* − Ω);
            Return n;
        End If
    End For
    Return INFEASIBLE;
```

Figure 4: Code assignment for Ω.

5 Simulation Results

5.1 Simulation Method

In this section, we present results obtained from a Matlab simulation of the proposed schemes. In the beginning of the simulation, the MHs are uniformly distributed in a single cell covering a range of 100 to 1000 meters from the BS. Each MH moves at a randomly chosen speed toward or against the BS. The speed is uniformly distributed from 0 to 1 meter per second. There are 12 active MHs in the cell at all times. At the beginning of every time slot (the time slot is assumed to be 0.02 seconds), each MH has probability $p = 0.02$ of ending its current transmitting session and becoming inactive. For simplicity of simulation, we assume that when a MH becomes inactive another MH becomes active and starts a new session. The location and the speed of the new MH are chosen randomly. The speed of the MH is assumed to be constant during its session. We update the position and channel gain of all active MHs. The path loss is assigned based on the model described in [16].

$$g = \left[20\log_{10}\left(\frac{4\pi d_0}{\lambda}\right) + 10\left(a - bh_b + \frac{c}{h_b}\right)\log_{10}\left(\frac{d}{d_0}\right)\right] + \left[10x\sigma_\gamma \log_{10}\left(\frac{d}{d_0}\right) + y\mu_\sigma + yz\sigma_\sigma\right]$$

where d is distance between BS and MH in meters, h_b is the base station antenna height in meters, $d_0 = 100$ meters and l is the carriers wavelength. The parameter x, y and z are independent zero-mean Gaussian random variables of unit standard deviation, $N(0, 1)$. The parameter values used for this model are: the frequency of the carrier is 1.9 GHz, $h_b = 50$ meters, $a = 4.0$, $b = 0.0065$, $c = 17.1$, $\sigma_\gamma = 0.75$, $\mu_\sigma = 9.6$, and $\sigma_\sigma = 3.0$.

For simplicity, we use linear service curves for all MHs. The slope of MH$_i$'s service curve, R_i, is its desired data rate. We assume $\forall i : M_i = 10$, the processing gain $G = 120$, and the desired SINR $\gamma = 8$dB. The data rate and the maximum transmit powers are varied in the simulation to study their effect on the performance of the proposed scheme.

The simulation is run for a long period time T. For results presented here $T = 1,000$ seconds. At the end of a time slot, say at time t, the simulator collects two measures: System Throughput $\Omega(t)$ and the Normalized System Deviation $\theta(t)$. These two measures are formally defined as follows.

$$\text{System Throughput} \qquad \Omega(t) = \sum_{i=1}^{K} n_i$$

$$\text{Normalized System Deviation} \qquad \theta(t) = \frac{\sum_{i=1}^{K} \max\{0, D_i(t) - W_i(t)\}}{\sum_{i=1}^{K} D_i(t)}$$

The graphs in this section plot the average of $\Omega(t)$ and $\theta(t)$ computed over the entire simulation run. That is,

$$\text{Average Throughput} \qquad \bar{\Omega} = \frac{\Delta}{T} \sum_{l=1}^{T/\Delta} \Omega(l\Delta)$$

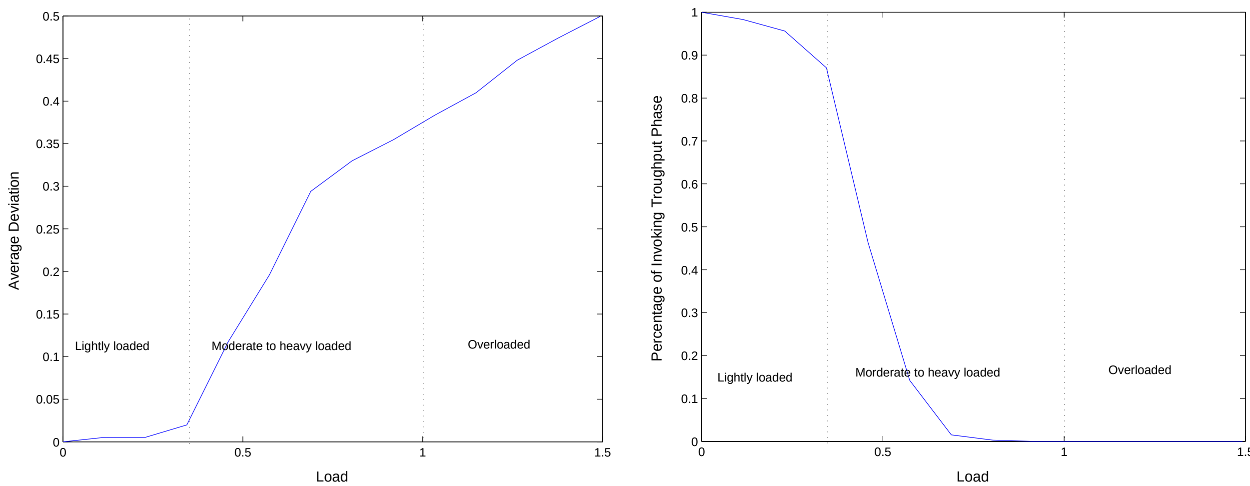
$$\text{Average Deviation} \qquad \bar{\theta} = \frac{\Delta}{T} \sum_{l=1}^{T/\Delta} \theta(l\Delta).$$

Let Ω^* be the average throughput achieved when $\sum_i R_i = 0$. Since $\sum_i R_i = 0$ only if $R_i = 0$ for all i, Ω^* is the throughput achieved when there are no service curve constraints. In the presence of additional service curve constraints, the throughput achieved must be less than or equal to Ω^*. That is, there is no way a system can meet all service curve constraints if $\sum_i R_i > \Omega^*$. We define the *load of the system* to be $\sum_i R_i/\Omega^*$.

5.2 Simulation Results

5.2.1 Deviation vs. Load

Figure 5(a) shows the average system deviation as a function of load for forward link. The results here correspond to a maximum transmit power 140 dB as compared to noise power. Figure 5(b) shows the percentage of time slots in which the proposed scheme invokes the throughput phase for different loads. The average deviation increases with load. The deviation is close to 0 when system is lightly loaded (loads less than 0.4). This is because the service curve can be met most of the time. This can also be seen in Figure 5(b) since the proposed scheme invokes throughput phase only in time instants where the service curve of all the MHs are met. For example, at loads of 0.3, the throughput phase is invoked 90% of the time slots, implying that the service curve of all MHs were met 90% of the time. When the system load is moderate to heavy (0.4 to 1), it is more difficult to meet the service curve of all MHs. As a result, the system deviation starts to increase. Finally,



(a) Deviation vs. load

(b) Percentage of throughput phase

Figure 5: Forward link.

when the system load exceeds 1, it is not possible to meet the service curve requirement almost always (see Figure 5(b)). The deviation therefore continues to increase.

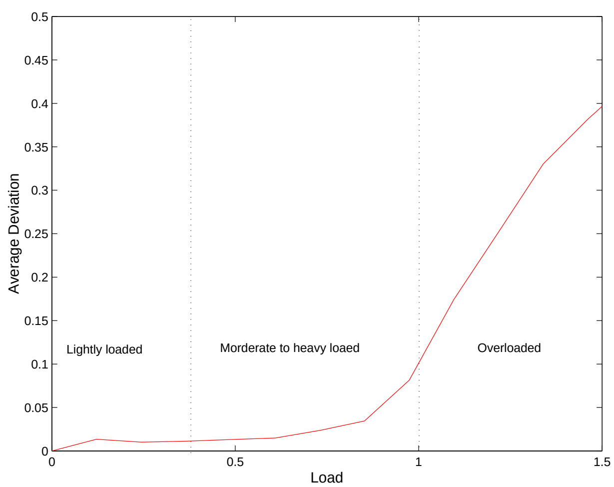
Similar results for reverse link are shown in Figures 6(a) and 6(b). The results here correspond to maximum transmit powers of 130 dB as compared to noise power. Although the numerical values differ between uplink and reverse link, the trends and the corresponding reasons are the same.

5.2.2 Throughput vs. Load

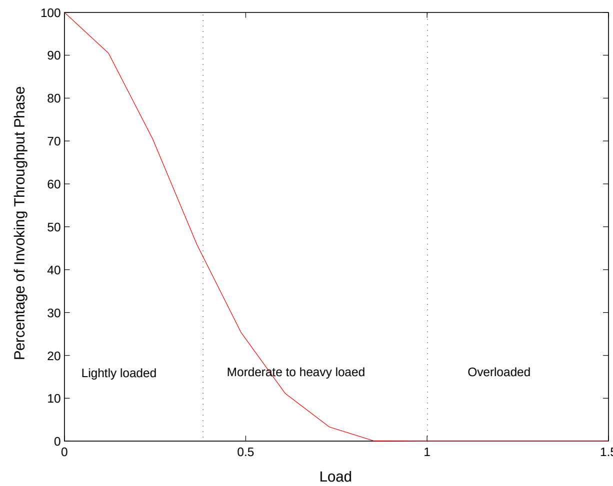
Figures 7(a) and 7(b) show the normalized system throughput for uplink and forward link respectively.

The following observations can be made.

1. The throughput decreases with the load when the system is lightly loaded (load $<$ 0.4). This is because the service curves requirement forces the scheme to allocate codes to MHs with poor channel. For forward link, a code allocated to a poor channel consumes more transmit power than a code assigned to a good channel. For the reverse link case, a code allocated to a poor channel cannot tolerate as much interference as a code allocated to a good channel. Consequently, for both forward and reverse links, there is a decrease in the total number of codes that can be assigned.

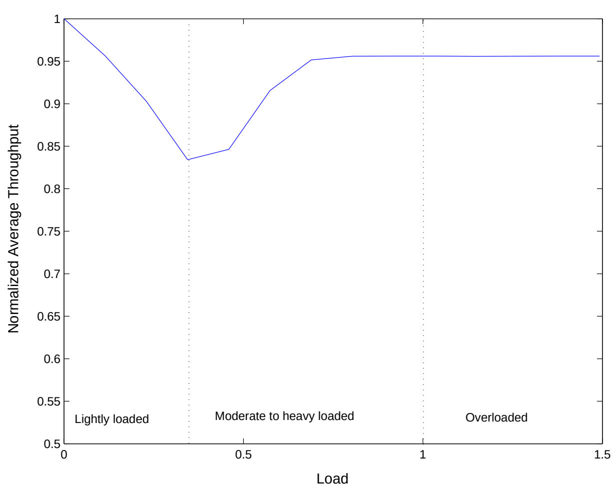


(a) Deviation vs. load

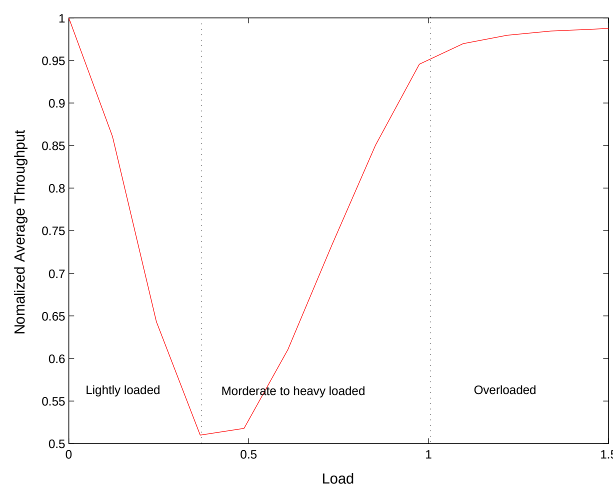


(b) Percentage of throughput phase

Figure 6: Reverse link.



(a) Forward link



(b) Reverse link

Figure 7: Throughput vs load.

2. The throughput increases with load when the system is moderate to fully loaded (0.4 to 1). This increase in throughout occurs at the expense of deviation (see Figures 5(a) and 6(a)). To better understand the reason for this, consider the case when system is almost fully loaded. In this case, most MHs are not able to meet their service curves. Therefore, m_i is large for almost all MH_i and $l_i = \min(m_i, M_i) = M_i$ for almost all i. Consequently, the deviation phase maximizes throughput subject to $0 < n_i < M_i$. Note, its similarity to the case when there is no service curve requirement (load = 0). In this case, $m_i = 0$ and thus $l_i = 0$ and the proposed scheme always uses the throughput phase subject to $0 < n_i < M_i$. As a result, the throughput achieved when load is close to 1 is almost as high as when there is no service curve requirement. (But the deviation is large). Therefore, the deviation phase of the proposed scheme is almost the same as the throughput phase without any service curve requirement.

3. The throughput saturates when load exceeds 1. This is because the maximum throughput achieved in this case cannot exceed the max achievable throughput (i.e., when there are no service curve requirements). The deviation, however, continues to increase. Since the throughput is saturated, the increase in deviation is linear with increase in load (see Figures 5(a) and 6(a)).

6 Conclusion

In this paper, we developed adaptive forward and reverse link schemes for power and rate allocation in a DS-CDMA network. Our solution re-allocates rate and transmit power once every Δ time units. The primary objective of the re-allocation at time t is to minimize the system deviation from service curve at time $t + \Delta$. The secondary objective is to maximize the total throughput when the primary objective can be completely met.

The performance results are similar for forward and reverse links. The results show that, under different system loads, the scheme will adjust tradeoffs between total system throughput and deviation automatically.

Appendix

A Proof of lemma 1

Since

$$\sum_i \max\{D_i(t+\Delta) - W_i(t+\Delta), 0\} = \sum_i \max\{[D_i(t+\Delta) - W_i(t)] - [W_i(t+\Delta) - W_i(t)], 0\}$$

and dividing the objective by a constant does not alter the optimal solution, minimizing $\sum_i \max\{D_i(t+\Delta) - W_i(t+\Delta), 0\}$ is equivalent to minimizing $\sum_i \max\left\{\frac{D_i(t+\Delta)-W_i(t)}{F} - n_i, 0\right\}$.

First, consider a subset S of MHs such that for each i in S, $D_i(t+\Delta) - W_i(t) > 0$. i.e.,

$$S \equiv \{i : D_i(t+\Delta) - W_i(t) > 0\},$$

Let

$$V(n) \equiv \sum_{i\in S} \max\left\{\frac{D_i(t+\Delta) - W_i(t)}{F} - n_i, 0\right\}$$

and

$$U(n) \equiv \sum_{i\in S} \max\left\{\left\lceil\frac{D_i(t+\Delta) - W_i(t)}{F}\right\rceil - n_i, 0\right\}$$

Let $\tilde{n}$ be an optimal solution that minimizes $V(n)$ subject to the rate, power and SINR constraints and $0 \le n_i \le m_i$ for all i. We show using the method of contradiction that $\tilde{n}$ also minimizes $U(n)$. Suppose $\tilde{n}$ not minimize $U(n)$. Then, there must exist a n^* such that $V(\tilde{n}) < V(n^*)$ and $U(\tilde{n}) > U(n^*)$. However this is contradict because

$$\begin{aligned}
& V(\tilde{n}) < V(n^*) \\
\Leftrightarrow\ & \sum_{i\in S} \max\left\{\frac{D_i(t+\Delta) - W_i(t)}{F} - \tilde{n}_i, 0\right\} < \sum_{i\in S} \max\left\{\frac{D_i(t+\Delta) - W_i(t)}{F} - n_i^*, 0\right\} \\
\Leftrightarrow\ & \sum_{i\in S} \max\left\{\left\lceil\frac{D_i(t+\Delta) - W_i(t)}{F}\right\rceil - \alpha_i - \tilde{n}_i, 0\right\} < \sum_{i\in S} \max\left\{\left\lceil\frac{D_i(t+\Delta) - W_i(t)}{F}\right\rceil - \alpha_i - n_i^*, 0\right\} \\
\Leftrightarrow\ & \sum_{i\in S} \max\left\{\left\lceil\frac{D_i(t+\Delta) - W_i(t)}{F}\right\rceil - \tilde{n}_i, 0\right\} < \sum_{i\in S} \max\left\{\left\lceil\frac{D_i(t+\Delta) - W_i(t)}{F}\right\rceil - n_i^*, 0\right\} \\
\Leftrightarrow\ & U(\tilde{n}) < U(n^*)
\end{aligned}$$

where $\alpha_i = \left\lceil \frac{D_i(t+\Delta)-W_i(t)}{F} \right\rceil - \frac{D_i(t+\Delta)-W_i(t)}{F}$.

Therefore, $\tilde{n}$ also minimizes $U(n)$. Further, since

$$\sum_{i \notin S} \max\left\{ \frac{D_i(t+\Delta)-W_i(t)}{F} - \tilde{n}_i, 0 \right\} = 0 = \sum_{i \notin S} \max\left\{ \left\lceil \frac{D_i(t+\Delta)-W_i(t)}{F} \right\rceil - n_i, 0 \right\}$$

$\tilde{n}$ that minimizes

$$V(n) + \sum_{i \notin S} \max\left\{ \frac{D_i(t+\Delta)-W_i(t)}{F} - n_i, 0 \right\}$$

also minimizes

$$U(n) + \sum_{i \notin S} \max\left\{ \left\lceil \frac{D_i(t+\Delta)-W_i(t)}{F} \right\rceil - n_i, 0 \right\}$$

Due to rate constraint, $n_i \leq M_i$. Moreover, the deviation $\max\{D_i(t+\Delta)-W_i(t+\Delta), 0\}$ for MH_i is equal to zero for all $n_i \geq m_i$. Therefore, to minimize the objective function is not necessary to assign $n_i > m_i$. From these two observations it follows that imposing the additional constraints $0 \leq n_i \leq \min\{m_i, M_i\} = l_i$ for all i does not alter the optimal solution. Hence, the lemma 1 holds. ■

B Proof of Lemma 5

Let Ω_1 be an optimal solution of the first problem and Ω_2 be an optimal solution of the second problem.

First we show that Ω_1 satisfies constraints of the second problem.

The condition of invoking throughput phase is that for all i, $n_i \geq l_i$. Thus

$$\forall i: \quad \frac{l_i R_C(\Omega_1)}{g_i} \leq \frac{n_i R_C(\Omega_1)}{g_i} \leq P_{i,\max} \tag{B.22}$$

Applying solution of (16), we can get condition (i) in the second problem. This means that Ω_1 also satisfies constraint (i) in the second problem.

Ω_1 satisfies constraint (iii) in the first problem directly implies that it satisfies constraint (ii) in the second problem.

Let $\tilde{n}_i$ denote the maximum possible number of spreading codes for MH_i when throughput is Ω, $\tilde{n}_i = \left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega)} \right\rfloor$. A feasible code assignment $\tilde{\mathbf{n}}$ implies that

$$\Omega_1 \leq \sum_i \tilde{n}_i = \sum_i \left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega)} \right\rfloor$$

Because Ω_1 satisfies constraint (iii) of the first problem, it also satisfies constraint (iii) in the second problem.

Therefore,

$$\Omega_2 \geq \Omega_1 \tag{B.23}$$

Second, we show that Ω_2 satisfies constraints of the first problem.

Given Ω_2 as an optimal solution of the second problem. Define n_i by (21). It is clear that $\sum_i n_i = \Omega_2$. We now show that n_i satisfies the constraints of the second problem.

(B.22) and constraint (i) of the second problem can promise that $\left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega_2)} \right\rfloor \geq l_i$ and $x_i \geq 0$.

From three constraints of the second problem, we know that the lower bound of throughput Ω_2 is $\sum_i l_i$. The above assignment meets this requirement.

The lowest upper bound of Ω_2 is the minimum of the following three terms, $\sum_i M_i$, $\sum_i \left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega_2)} \right\rfloor$ and $\frac{G+\gamma}{\gamma} - \frac{\sigma^2 l_i}{g_i P_{i,\max}}$. This assignment can also meet these constraints.

Next, we show that Ω_2 satisfies constraints of the first problem.

From the assignment, we known that

$$\forall i: \quad 0 \leq x_i \leq \min\{M_i, \left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega_2)} \right\rfloor\} - l_i$$

Therefore,

$$l_i \leq n_i \leq \min\{M_i, \left\lfloor \frac{g_i P_{i,\max}}{R_C(\Omega_2)} \right\rfloor) \tag{B.24}$$

According to (16), $\frac{n_i R_C(\Omega_2)}{g_i} \leq P_{i,\max}$ must hold in order to meet SINR constraint. From (B.24), we know that it is true. This shows that power constraint (constraint (i)) of the first problem can also be met. Thus Ω_2 meets constraints (i) and (ii) in the first problem.

It is clear that this assignment meet constraint (iii) of the first problem $l_i \leq n_i \leq M_i$.

Therefore,

$$\Omega_1 \geq \Omega_2 \tag{B.25}$$

The lemma follows from (B.23) and(B.25)

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