Sufficient Conditions for Flow Admission Control in Wireless Ad-hoc Networks *

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Wireless nodes within the same vicinity contend with each other for accessing the shared wireless medium. The contention constraints for sharing the medium depend on the medium access control (MAC) protocol that administers the accessibility of the network. In a previous work, we derived a set of sufficient conditions under which rates of all the flows are feasible given a MAC protocol. In this paper, we derive a new set of sufficient conditions that are better than those developed earlier. We show that these conditions are useful in solving many problems such as linear programming-based routing and admission control for QoS routing. We also evaluate, compare and discuss the performance of the derived sufficient conditions through simulation studies.

I. Introduction

Wireless ad hoc networks consist of many self-organizing nodes which cooperatively maintain connectivity among each other without any need of a wired infrastructure or a centralized administration. Examples of such networks are sensor and Bluetooth networks. In these wireless networks, nodes within the same vicinity contend for the shared wireless medium. For example, in IEEE 802.11 medium access control (MAC) protocol-based networks, if a node i is communicating with a neighboring node j, then all nodes within the communication range of node i or j must remain idle. On the other hand, if nodes within a transmission range are allowed to use different frequencies (e.g., FDMA) or different spreading codes (e.g., CDMA), then two neighboring nodes can simultaneously use the wireless medium. Flows in these networks which do not share nodes can communicate at the same time. Furthermore, in networks where nodes are equipped with two radios (e.g., WINS networks [7, 13]), a node can transmit and receive simultaneously. Therefore, one can conclude that the achievable flow rates and/or the number of admitted flows in such networks depend on the physical capabilities of the wireless nodes and the medium contention resulting from the MAC protocol.

MAC contention constraints are critical in many networking problems [1–5, 8–12, 14–18]. For instance, linear programming-based routing formulations for wireless networks (e.g., sensor and Bluetooth networks) require MAC contention constraints to guarantee the physical feasibility of the resulting routing solutions [1, 2, 11].

In [8], wired link contention constraints are used as flow constraints to find the optimal system's rates. An upper bound on feasible rates is derived in [4] and used in [17] as a means of guaranteeing the feasibility of schedules. The only MAC constraint imposed on the medium considered in [4, 17] is that a node can either transmit or receive at a time. They consider networks that consist of nodes that, for example, use different frequencies or different codes if they are within each other's vicinity. In the remainder of this paper, we will refer to this MAC as MAC of [17]. However, the upper bound derived in [4] cannot be applied to CSMA/CA (e.g., IEEE 802.11) MAC-based wireless networks. Unlike MAC of [17], CSMA/CA MAC-based networks require more complex conditions to assure physical feasibility of rates [5].

MAC contention constraints also present a major challenge to QoS routing in ad hoc networks [3, 9, 10, 12, 14–16, 18]. Admission control and bandwidth calculation mechanisms of QoS routes require complete knowledge of not only a node's information but also its neighbors' information. The work in [9, 10] presents a bandwidth calculation heuristic algorithm for time-slotted (i.e., TDMA) networks. Under the assumption that nodes know their free transmission slots, the heuristic allows calculation, reservation, and scheduling of time slots (bandwidth) along already known routes between source-destination pairs of nodes. A similar bandwidth calculation heuristic for

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TDMA MAC-based networks is also proposed in [18]. Although these heuristics can be used in time-slotted networks, they cannot be used in CSMA/CA MAC-based networks to calculate available bandwidths. Because of the lack of adequate admission tests, QoS flows are difficult to assure in these networks. As a consequence, soft QoS algorithms are developed instead [15, 16]. In soft QoS, bandwidth-sensitive flows can be only given higher priority over best-effort flows, not guaranteed bandwidths. Typically, soft QoS is assured by letting QoS-sensitive flows adopt smaller contention window sizes than those adopted by the best-effort flows.

In a previous work [5] of ours, we derived a set of sufficient conditions under which flow rates are feasible in the sense that once satisfied, these rates are achievable by the given MAC protocol. In this paper, we derive better sufficient conditions than those derived earlier in [5]. We show that the proved conditions apply to a wide range of MAC protocols including IEEE 802.11, TDMA, and MAC of [17]. We show and argue that these conditions are useful in solving many problems such as linear programming-based routing problems and admission control mechanisms for QoS routing. Through illustrative examples, we show the importance of including the physical contention constraints by means of the proved sufficient conditions in determining routing solutions and admitting new flows into the network. We evaluate and compare the performance of the proved conditions through simulation studies. Based on the obtained numerical results, we discuss the applications of these conditions.

The paper is organized as follows. Section II presents the network model. In Section III, we prove sufficient and necessary conditions under which flow data rates are feasible. Simulation results showing and comparing the performance of the proved sufficient conditions are given in Section IV. Finally, we conclude the paper in Section V.

II. Model

We model the wireless ad hoc network as an undirected network graph $G = (\mathcal{N}, \mathcal{L})$ with a finite non-empty set $\mathcal{N}$ of nodes and a set of wireless links $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$. $\mathcal{L}$ is the set of all pairs (n, m) of distinct nodes in $\mathcal{N}$ such that n and m are within each other's transmission range. An ordered pair of nodes (n, m) in $\mathcal{L}$ is said to form a flow[1] f if n needs to transmit to m. The flow f is said to be *active* if n is

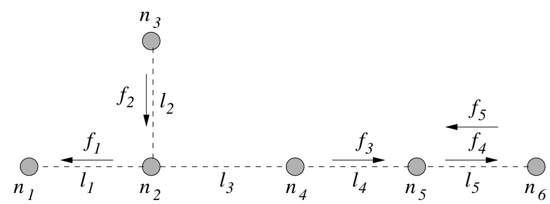


Figure 1: Network graph for Example 1.

currently transmitting to m otherwise the flow is said to be *inactive*. If two flows f and g cannot be active at the same time, then we say that f and g contend with each other.

Let $\mathcal{F}$ denote the set of all flows in G. We model the set of flows $\mathcal{F}$ as an undirected graph $H = (\mathcal{F}, \mathcal{C})$ where $\mathcal{C}$ is the set of all distinct contending pairs of flows in $\mathcal{F}$. Note that the graph H, referred to as *flow contention* graph, depends mainly on (i) the network graph G (i.e., node connectivity), (ii) the set of flows $\mathcal{F}$ (i.e., routes generated by nodes), and (iii) the MAC protocol (i.e., medium access constraints imposed on the set of flows).

For every $f \in \mathcal{F}$, let Ψ_f be the set of flows in $\mathcal{F}$ contending with f; i.e., $\Psi_f = \{g \in \mathcal{F} : (f, g) \in \mathcal{C}\}$. It is worth noting that two flows belonging to the same set Ψ_f of a flow f could be active simultaneously. However, no flow $g \in \Psi_f$ can be active simultaneously with f. For every $f \in \mathcal{F}$, let d_f denote[2] $|\Psi_f|$. We call the set Ψ_f and the number d_f respectively the *contention set* and the *degree* of flow f.

Example 1 *In this example, we determine the flow contention graphs of a given network graph operating under two different MACs: MAC of IEEE 802.11 and MAC of [17]. Consider the network* $G = (\mathcal{N}, \mathcal{L})$ *shown in Fig. 1 where* $\mathcal{N} = \{n_1, n_2, n_3, n_4, n_5, n_6\}$ *and* $\mathcal{L} = \{l_1, l_2, l_3, l_4, l_5\}$. *In the figure, a dashed line (e.g.,* l_3*) between a pair of nodes (e.g.,* (n_2, n_4)*) implies that these nodes are within each other's communication range. Consider the set of flows* $\mathcal{F} = \{f_1, f_2, f_3, f_4, f_5\}$ *as shown in Fig. 1. A flow f (e.g.,* f_4*) is formed by an ordered pair of nodes* (n, m) *(e.g.,* (n_5, n_6)*) if* n *(e.g.,* n_5*) needs to transmit to* m *(e.g.,* n_6*). In the IEEE 802.11 MAC-based network, a node can either transmit or receive at any time, and if node* n *is transmitting to node* m*, then all nodes within the communication range of* n *or* m *must remain idle. On the other hand, under MAC of [17], the only restriction is that a node can communicate with at most one node at a time. Thus, flows which do not share nodes can be active simultaneously. The flow con-*

[1] In this paper, the term flow refers to one-hop flow.

[2] $|X|$ denotes the cardinality of the set X.

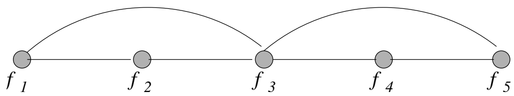


Figure 2: Flow contention graph for Example 1: MAC of IEEE 802.11.

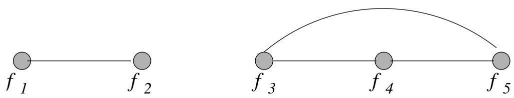


Figure 3: Flow contention graph for Example 1: MAC of [17].

tention graphs $H_1 = (\mathcal{F}, \mathcal{C}_1)$ and $H_2 = (\mathcal{F}, \mathcal{C}_2)$ of the network G respectively operating under MAC of IEEE 802.11 and MAC of [17] are given in Figs. 2 and 3. The contention sets and the degrees of the flows in H_1 and H_2 are given in Table 1. ■

Given a network $G = (\mathcal{N}, \mathcal{L})$, the flow contention graph $H = (\mathcal{F}, \mathcal{C})$ can always be derived for both MACs—MAC of IEEE 802.11 and MAC of [17].

III. Conditions for Rate Feasibility

In this section, we use the flow contention graph as a means of representing the wireless ad hoc network. Consequently, the results apply to all wireless ad hoc networks for which the flow contention graph can be derived; e.g., IEEE 802.11-based networks.

Let $H = (\mathcal{F}, \mathcal{C})$ be a flow contention graph. Let us assume that each flow f in $\mathcal{F}$ flows data traffic at a rate of x_f bits per second. Let $\mathbf{x} = (x_f)_{f \in \mathcal{F}}$ be the vector, referred to as *flow rate vector*, representing the data rates of all flows in $\mathcal{F}$. The vector $\mathbf{x}$ is said to be *feasible* flow rate vector in H if there exists a time schedule in which the rates of all flows are satisfied. Formally, $\mathbf{x}$ is feasible in H if there exists a time schedule $S = [0, \tau]$ of length $\tau > 0$ in which every flow $f \in \mathcal{F}$ communicates τx_f bits. For each subset of flows $\mathcal{A} \subseteq \mathcal{F}$, we define the *weight* of $\mathcal{A}$ under a given flow rate vector $\mathbf{x}$ to be $\delta(\mathcal{A}, \mathbf{x}) = \sum_{f \in \mathcal{A}} x_f$. Let C denote the link bandwidth of the wireless medium; i.e., a maximum of τC bits can be transmitted in the interval $[0, \tau]$.

Table 1: Contention sets and degrees of flows for Example 1.

flow f	MAC of IEEE 802.11 Ψ_f	d_f	MAC of [17] Ψ_f	d_f
f_1	$\{f_2, f_3\}$	2	$\{f_2\}$	1
f_2	$\{f_1, f_3\}$	2	$\{f_1\}$	1
f_3	$\{f_1, f_2, f_4, f_5\}$	4	$\{f_4, f_5\}$	2
f_4	$\{f_3, f_5\}$	2	$\{f_3, f_5\}$	2
f_5	$\{f_3, f_4\}$	2	$\{f_3, f_4\}$	2

In this section, we prove sufficient conditions under which a given flow rate vector is feasible in H. For completeness, we first provide the set of sufficient conditions that we already derived in [5]. This set depends on the topology's parameters of the flow contention graph as well as the rates of flows. Then, we present another set of sufficient conditions which are based on topology's parameters only. Finally, we present and prove our main theorem that states a better set of sufficient conditions than the previous two sets. Hereafter, $\mathbf{x} = (x_f)_{f \in \mathcal{F}}$ denotes a flow rate vector in H.

Theorem 1 $\mathbf{x}$ *is feasible in H if $x_f \leq C - \delta(\Psi_f, \mathbf{x})$ for all $f \in \mathcal{F}$. The set of these conditions will be referred to as* Rate Feasibility Test *(*`RateFT`*).*

Proof: Let N denote $|\mathcal{F}|$ and $\mathbf{x} = (x_f)_{f \in \mathcal{F}}$ be a flow rate vector such that $x_f \leq C - \delta(\Psi_f, \mathbf{x})$ for all $f \in \mathcal{F}$. Without loss of generality, let us arrange the flows in $\mathcal{F}$ as $\{1, 2, \ldots, N\}$ such that $x_i \leq x_j$ for all $1 \leq i \leq j \leq N$, and let $\mathcal{F}^i$ denote the set of flows $\{1, 2, \ldots, i\}$. Let $S = [0, \tau]$ be a time schedule of length $\tau > 0$ seconds. We show that for all $n = 1, 2, \ldots, N$, the flows in the subset $\mathcal{F}^n \subseteq \mathcal{F}$ are schedulable in S. (Thus, $\mathcal{F} \equiv \mathcal{F}^N$ is schedulable.) We proceed the proof by induction.
BASIS: $\mathcal{F}^1 = \{1\}$. Since $x_1 \leq C - \delta(\Psi_1, \mathbf{x})$, then $x_1 \leq C$ and consequently $\mathcal{F}^1$ is schedulable in S.
INDUCTION STEP: Assume that all the flows in $\mathcal{F}^{n-1}$ are schedulable in S for any n, $1 < n \leq N$, and show that the flows in $\mathcal{F}^n$ are also schedulable in S. Since $\mathcal{F}^n = \mathcal{F}^{n-1} \cup \{n\}$, then it suffices to prove that flow n can be scheduled, provided that all flows in $\mathcal{F}^{n-1}$ are already scheduled. Let Φ_n be the set of flows in $\mathcal{F}^{n-1}$ that contend with flow n; i.e., $\Phi_n = \mathcal{F}^{n-1} \cap \Psi_n$. Since $\Phi_n \subseteq \Psi_n$ and $x_n \leq C - \delta(\Psi_n, \mathbf{x})$, then $x_n \leq C - \delta(\Phi_n, \mathbf{x})$. Therefore, even if all the flows in Φ_n (the only previously scheduled flows that contend with n) were already scheduled disjointly in $\mathcal{F}^{n-1}$, flow n can still be scheduled in S. ■

Theorem 2 $\mathbf{x}$ *is feasible in H if $x_f \leq \frac{C}{d_f + 1}$ for all $f \in \mathcal{F}$. The set of these conditions will be referred to as* Degree Feasibility Test *(*`DegreeFT`*).*

Proof: Let N denote $|\mathcal{F}|$ and $\mathbf{x} = (x_f)_{f \in \mathcal{F}}$ be a flow rate vector such that $x_f \leq \frac{C}{d_f + 1}$ for all $f \in \mathcal{F}$. Similar to the proof of Theorem 1, let us again arrange the flows in $\mathcal{F}$ as $\{1, 2, \ldots, N\}$ such that $x_i \leq x_j$ for all

$1 \leq i \leq j \leq N$, and let $\mathcal{F}^i$ denote $\{1, 2, \ldots, i\}$. Let $S = [0, \tau]$ be a time schedule of length $\tau > 0$ seconds. We show by induction that for all $n = 1, 2, \ldots, N$, the flows in the subset $\mathcal{F}^n \subseteq \mathcal{F}$ are schedulable in S.

BASIS: $\mathcal{F}^1 = \{1\}$. Since $d_1 \geq 0$ and $x_1 \leq \frac{C}{d_1+1}$, then $x_1 \leq C$ and thus $\mathcal{F}^1$ is schedulable in S.

INDUCTION STEP: Assume that all the flows in $\mathcal{F}^{n-1}$ are schedulable in S for any n, $1 < n \leq N$, and show that the flows in $\mathcal{F}^n$ are also schedulable in S. Since $\mathcal{F}^n = \mathcal{F}^{n-1} \cup \{n\}$, then it suffices to prove that flow n can be scheduled, provided that all flows in $\mathcal{F}^{n-1}$ are already scheduled. Let Φ_n be the set of flows in $\mathcal{F}^{n-1}$ that contend with flow n; i.e., $\Phi_n = \mathcal{F}^{n-1} \cap \Psi_n$. Note that for all $k \in \Phi_n$, $x_k \leq x_n$ (since flow k is already scheduled). Hence, since $x_n \leq \frac{C}{d_n+1}$, then $x_k \leq \frac{C}{d_n+1}$ for all $k \in \Phi_n$. Therefore, each flow k (contending with n and already scheduled in $\mathcal{F}^{n-1}$) needs at most a fraction $\frac{1}{d_n+1}$ of the total schedule. (A fraction $\frac{1}{d_n+1}$ in S corresponds to a period of $\frac{\tau}{d_n+1}$ seconds during which k flows $C \times \frac{\tau}{d_n+1}$ bits which are greater than or equal to the required $x_k \times \tau$ bits since $x_k \leq \frac{C}{d_n+1}$.) The worst case occurs when all of the d_n flows that contend with flow n happen to be in $\mathcal{F}^{n-1}$ (i.e., $|\Phi_n| = d_n$) and also scheduled disjointly. In such a case, they will occupy a fraction of at most $d_n \times \frac{1}{d_n+1}$ of the total time schedule. This leaves us with a fraction of at least $\frac{1}{d_n+1}$ which is exactly what is needed for flow n to be schedulable in S without overlapping with any of its contending flows. ■

One point that requires attention is that none of the two sets of sufficient conditions (i.e., `RateFT` and `DegreeFT`) is better than the other. That is, flow rate vectors satisfying the conditions in `DegreeFT` do not necessarily satisfy those in `RateFT` and vise versa. This is illustrated by the following example.

Example 2 *Again, let us consider the network G given in Fig. 1 where IEEE 802.11 is the medium access control protocol, and whose flow contention graph is given in Fig. 2. Let $C = 1$. Consider the flow rate vectors $\mathbf{x}'$ and $\mathbf{x}''$ such that $\mathbf{x}' = (\frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{4}, \frac{1}{4})$ and $\mathbf{x}'' = (\frac{1}{8}, \frac{1}{8}, \frac{1}{2}, \frac{1}{8}, \frac{1}{8})$. In Tables 2 and 3, we show numerical verifications respectively for the vectors $\mathbf{x}'$ and $\mathbf{x}''$. Note that $\mathbf{x}'$ satisfies the conditions in* `DegreeFT` *(Theorem 2) and $\mathbf{x}''$ satisfies the conditions in* `RateFT` *(Theorem 1). However, $\mathbf{x}'$ does not satisfy the sufficient conditions in* `RateFT` *since $x'_{f_3} = \frac{1}{8} > C - \delta(\Psi_{f_3}, \mathbf{x}') = 0$. Likewise, $\mathbf{x}''$, which satisfies the conditions in* `RateFT` *, does not meet the required conditions in* `DegreeFT` *since $x_{f_3} = \frac{1}{2}$ exceeds $\frac{C}{d_{f_3}+1} = \frac{1}{5}$.* ■

Table 2: Numerical verification under $\mathbf{x}'$ for Example 2.

flow f	$\frac{C}{d_f+1}$	x'_f	$C - \delta(\Psi_f, \mathbf{x}')$
f_1	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{5}{8} = 1 - \frac{1}{4} - \frac{1}{8}$
f_2	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{5}{8} = 1 - \frac{1}{4} - \frac{1}{8}$
f_3	$\frac{1}{5}$	$\frac{1}{8}$	$0 = 1 - 4 \times \frac{1}{4}$
f_4	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{5}{8} = 1 - \frac{1}{8} - \frac{1}{4}$
f_5	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{5}{8} = 1 - \frac{1}{8} - \frac{1}{4}$

Table 3: Numerical verification under $\mathbf{x}''$ for Example 2.

flow f	$\frac{C}{d_f+1}$	x''_f	$C - \delta(\Psi_f, \mathbf{x}'')$
f_1	$\frac{1}{3}$	$\frac{1}{8}$	$\frac{3}{8} = 1 - \frac{1}{8} - \frac{1}{2}$
f_2	$\frac{1}{3}$	$\frac{1}{8}$	$\frac{3}{8} = 1 - \frac{1}{8} - \frac{1}{2}$
f_3	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{2} = 1 - 4 \times \frac{1}{8}$
f_4	$\frac{1}{3}$	$\frac{1}{8}$	$\frac{3}{8} = 1 - \frac{1}{2} - \frac{1}{8}$
f_5	$\frac{1}{3}$	$\frac{1}{8}$	$\frac{3}{8} = 1 - \frac{1}{2} - \frac{1}{8}$

As illustrated by the above example, neither `RateFT` nor `DegreeFT` can be said to be a better sufficient condition than the other. In fact, from a mathematical standpoint, one cannot draw any conclusion stating that one condition performs better than the other in the sense that when used, for example, in an admission control mechanism, that best one would result in a higher acceptance rate than the other one. This performance comparison can, however, be done through simulations as shown in the last section. The following theorem presents a set of sufficient conditions that we prove to be better than those stated by `RateFT` and `DegreeFT`.

Theorem 3 $\mathbf{x}$ *is feasible in H if $x_f \leq \max\{\frac{C}{d_f+1}, C - \delta(\Psi_f, \mathbf{x})\}$ for all $f \in \mathcal{F}$. Hereafter, we will refer to the set of these sufficient conditions as* Mixed Feasibility Test *(*`MixedFT`*).*

Proof: The proof of the theorem also presents a scheduling algorithm. Suppose that $\mathbf{x} = (x_f)_{f \in \mathcal{F}}$ is such that $x_f \leq \max\{\frac{C}{d_f+1}, C - \delta(\Psi_f, \mathbf{x})\}$ for all $f \in \mathcal{F}$. Again, without loss of generality, let us denote $|\mathcal{F}|$ by N, arrange the flows in $\mathcal{F}$ as $\{1, 2, \ldots, N\}$ such that $x_i \leq x_j$ for all $1 \leq i \leq j \leq N$, and denote the set of flows $\{1, 2, \ldots, i\}$ by $\mathcal{F}^i$. Let $S = [0, \tau]$ be a time schedule of length $\tau > 0$ seconds. We show by induction that for all $n = 1, 2, \ldots, N$, the set $\mathcal{F}^n$ is schedulable in S.

BASIS: $\mathcal{F}^1 = \{1\}$. Since $d_1 \geq 0$ and $\delta(\Psi_1, \mathbf{x}) \geq 0$, $\frac{C}{d_1+1} \leq C$ and $C - \delta(\Psi_1, \mathbf{x}) \leq C$. Further, since from hypothesis $x_1 \leq \max\{\frac{C}{d_1+1}, C - \delta(\Psi_1, \mathbf{x})\}$,

then $x_1 \leq C$ and thus $\mathcal{F}^1$ is schedulable in S.
INDUCTION STEP: Assume that all the flows in $\mathcal{F}^{n-1}$ are schedulable in S for any n, $1 < n \leq N$, and show that the flows in $\mathcal{F}^n$ are also schedulable in S. Since $\mathcal{F}^n = \mathcal{F}^{n-1} \cup \{n\}$, then it suffices to prove that flow n can be scheduled, provided that all flows in $\mathcal{F}^{n-1}$ are already scheduled. Let Φ_n be the set of flows in $\mathcal{F}^{n-1}$ that contend with flow n. Recall that $\Phi_n \subseteq \Psi_n$. There are two cases to consider:
Case (i): $\frac{C}{d_n+1} \leq C - \delta(\Psi_n, \mathbf{x})$. Since $x_n \leq C - \delta(\Psi_n, \mathbf{x})$, then proving that flow n is schedulable follows from the proof of Theorem 1.
Case (ii): $\frac{C}{d_n+1} > C - \delta(\Psi_n, \mathbf{x})$. Since $x_n \leq \frac{C}{d_n+1}$, then proving that flow n is schedulable follows from the proof of Theorem 2. ■

It is worth noting that the set of conditions stated in `MixedFT` is always better than any of the two other sets, `RateFT` and `DegreeFT`, in the sense that all flow rate vectors that satisfy the conditions in either `RateFT` and/or `DegreeFT` also satisfy those in `MixedFT`. However, a vector satisfying the conditions in `MixedFT` does not necessarily satisfy those in `RateFT` or `DegreeFT`.

Example 3 *In this example, we show that the sufficient conditions in* `MixedFT` *(Theorem 3) are not necessary. Consider the network G studied in Example 1 (See Fig. 1). Assume IEEE 802.11-based medium access control protocol. That is, (i) a node can either transmit or receive at any time, and (ii) if node n is transmitting to node m, then all nodes within the same communication range of n or m must remain idle. Also, for simplicity, let C equal 1. The flow contention graph that corresponds to G under the IEEE 802.11 MAC is given in Fig. 2. Now, consider a flow rate vector $\mathbf{x}$ equal to $(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, \frac{1}{4}, \frac{1}{4})$. Table 4 provides numerical calculation needed to verify that the conditions in* `MixedFT` *are not necessary. Note that the vector $\mathbf{x}$ does not satisfy the sufficient conditions in* `MixedFT` *since $x_{f_3} = \frac{1}{2} > \max\{\frac{C}{d_{f_3}+1}, C - \delta(\Psi_{f_3}, \mathbf{x})\} = \frac{1}{5}$. However, the flow rate vector $\mathbf{x}$ is physically feasible under MAC of IEEE 802.11. Fig. 4 shows a feasible time schedule S for $\mathbf{x}$ during which contending flows are scheduled in different spots: Flow f_3 is allocated the first half of S; the pair (f_1, f_4) is allocated the third quarter of S; and the pair (f_2, f_5) is allocated the last quarter of S. Note that flows f_1 and f_4 (also f_2 and f_5) are scheduled concurrently since they do not contend with each other.* ■

An example of applications in which `MixedFT`, `DegreeFT`, and/or `RateFT` are useful is network rout-

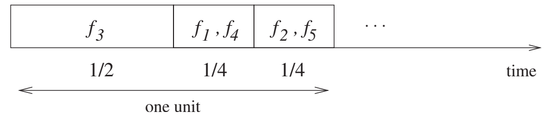


Figure 4: Time schedule for Example 3.

Table 4: Verification of Theorem 3 under $\mathbf{x}$ for Example 3.

flow f	Ψ_f	$C - \delta(\Psi_f, \mathbf{x})$	$\frac{C}{d_f+1}$
f_1	$\{f_2, f_3\}$	$1 - \frac{1}{4} - \frac{1}{2} = \frac{1}{4}$	$\frac{1}{3}$
f_2	$\{f_1, f_3\}$	$1 - \frac{1}{4} - \frac{1}{2} = \frac{1}{4}$	$\frac{1}{3}$
f_3	$\{f_1, f_2, f_4, f_5\}$	$1 - 4 \times \frac{1}{4} = 0$	$\frac{1}{5}$
f_4	$\{f_3, f_5\}$	$1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$	$\frac{1}{3}$
f_5	$\{f_3, f_4\}$	$1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$	$\frac{1}{3}$

ing. Routing problems are often formulated as linear programming problems (LPPs). For example, in [2], the problem of finding optimal routing solutions that maximize the network lifetime is formulated as an LPP. However, contention constraints resulting from the shared medium are not accounted for in this LPP formulation. As a result, rate solutions may be infeasible in the sense that the MAC protocol cannot satisfy the required rates. In the following example, we illustrate the importance of including the medium contention constraints (e.g., those in `RateFT`) in formulating these routing problems.

Example 4 *Consider a network $G = (\mathcal{N}, \mathcal{F})$ consisting of a set of 4 nodes $\mathcal{N} = \{n_1, n_2, n_3, n_4\}$, and a set of 4 flows $\mathcal{F} = \{f_1, f_2, f_3, f_4\}$ as shown in Fig. 5(a). Assume that node n_1 cannot directly communicate with node n_3 and node n_2 cannot directly communicate with node n_4. To demonstrate the significance of including the MAC constraints, we solve the same linear programming formulation given in [2] twice: without contention constraints (as defined in [2]), and with contention constraints (by including the sufficient conditions in* `RateFT`*). Let B_i be the initial amount of energy in Joules of n_i's battery. Assume that $B_1 = B_3 = B_4 = 1$ and $B_2 = 0.5$. Let $\epsilon = 0.01$ be the amount of energy in Joules needed to transmit one bit over a wireless link. Assume that the capacity C of the medium is 1 bit per second.*

Now, suppose that the only traffic in the network is from node n_1 to node n_3 at a rate of $\frac{2}{3}$ bit per second. The LPP in [2] is to find a flow rate vector $\mathbf{x} = (x_g)_{1 \leq g \leq 4}$ that maximizes the least remaining energy of all nodes over a period T seconds subject to flow balance constraints and energy consumption constraints (refer to [2] for more details). Assume that

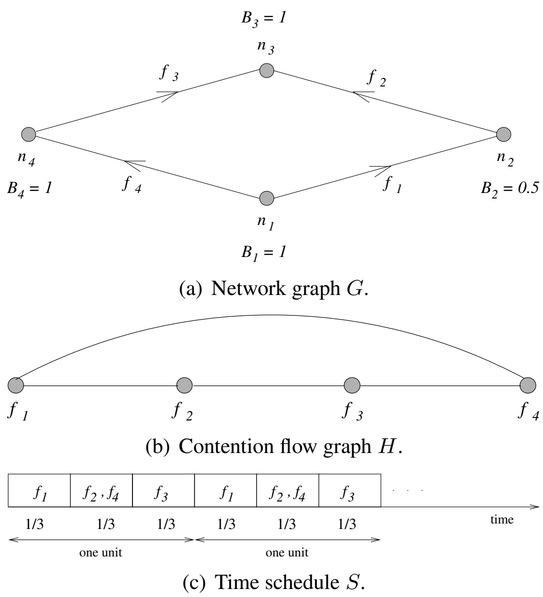


Figure 5: Graphs and time schedule for Example 4.

the period T is 1 second. The flow balance constraints as cited in [2] are:

$$\begin{aligned} 2/3 &= x_1 + x_4, \\ x_1 &= x_2, \\ x_2 + x_3 &= 2/3, \\ x_4 &= x_3, \\ x_g &\geq 0, \quad \forall\, g = 1,2,3, \textit{ or } 4. \end{aligned}$$

The energy consumption constraints state that the energy consumed by each node after T seconds is less than or equal to the initial amount of available energy.

One can verify easily that this LPP formulation has a unique optimal solution $\mathbf{x} = (0, 0, \frac{2}{3}, \frac{2}{3})$. The feasibility of this solution, however, depends on the MAC protocol. For instance, consider MAC of [17] where each node can communicate with at most one node at a time. Since the MAC protocol of [17] does not allow a node to receive and transmit at the same time, then flows f_3 and f_4 cannot be active simultaneously. As a result, a solution in which flows f_3 and f_4 both flow at a rate greater than the medium capacity of the wireless channel is not physically feasible under this MAC. Hence, the above solution $\mathbf{x}$ is not feasible because $x_3 + x_4 = \frac{2}{3} + \frac{2}{3}$ exceeds the capacity of the medium of 1 bit per second.

However, solving the linear programming formulation defined in [2] with the inclusion of the MAC constraints stated by the conditions in `RateFT` *results in a solution that is feasible. Fig. 5(b) shows the contention flow graph H of the network G under MAC of [17]. Specifically, the constraints given by the conditions in* `RateFT` *(Theorem 1) are:*

$$\begin{aligned} x_1 &\leq 1 - x_2 - x_4, \\ x_2 &\leq 1 - x_1 - x_3, \\ x_3 &\leq 1 - x_2 - x_4, \\ x_4 &\leq 1 - x_1 - x_3, \end{aligned}$$

and the optimal solution to the new LPP is $\mathbf{x}' = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. Note that, unlike $\mathbf{x}$, $\mathbf{x}'$ is now feasible under MAC of [17]. Fig. 5(c) shows a feasible time schedule S during which flows f_2 and f_4 are scheduled simultaneously. ■

The above example shows the importance of including the MAC contention constraints by means of `RateFT` *to routing formulations and their effect on the feasibility of rate solutions.*

Theorem 4 *If $x_f = x_g$ for all $f, g \in \mathcal{F}$, then* `RateFT`, `DegreeFT` *and* `MixedFT` *are equivalent.*

Proof: Let $\mathbf{x} = (x_f)_{f \in \mathcal{F}}$ be a flow rate vector in H and $r = x_f$ for all $f \in \mathcal{F}$. Note that $\delta(\Psi_f, \mathbf{x}) = r \times d_f$ for all $f \in \mathcal{F}$. Let $g \in \mathcal{F}$. 1) `RateFT` $\Leftrightarrow$ `DegreeFT`: $r \leq C - \delta(\Psi_g, \mathbf{x}) \Leftrightarrow r \leq C - r \times d_g \Leftrightarrow r \leq \frac{C}{d_g+1}$. 2) `MixedFT` $\Rightarrow$ `DegreeFT`: If $\max\{\frac{C}{d_g+1}, C - \delta(\Psi_g, \mathbf{x})\} = \frac{C}{d_g+1}$, then 2) is trivial. Else, 2) follows from 1). 3) `DegreeFT` $\Rightarrow$ `MixedFT`: 3) is trivial. ■

Theorem 5 *If $H = (\mathcal{F}, \mathcal{C})$ is a union of complete graphs, then the sufficient conditions in*

1. `MixedFT` *are necessary.*
2. `DegreeFT` *are not necessary.*
3. `RateFT` *are necessary.*

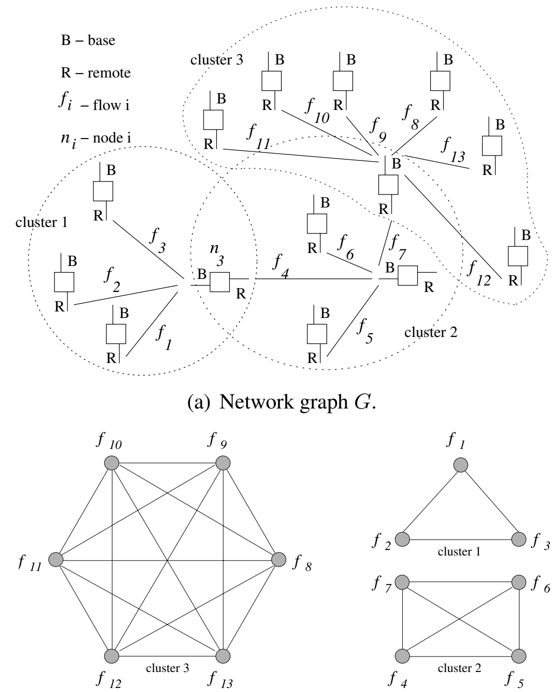
Proof: Let $\mathbf{x} = (x_f)_{f \in \mathcal{F}}$ be a feasible flow rate vector in H where H is a union of complete graphs. Let f be any flow in H, and $H_1 = (\mathcal{F}_1, \mathcal{C}_1)$ denote the complete graph to which f belongs. We proceed the proof is this order: 3), 1), and 2).
3) : We need to show that $x_f \leq C - \delta(\Psi_f, \mathbf{x})$. Note that since H_1 is complete, $\mathcal{F}_1 = \Psi_f \cup \{f\}$; which implies that $\sum_{g \in \mathcal{F}_1} x_g = \delta(\Psi_f, \mathbf{x}) + x_f$. Since $\mathbf{x}$ is feasible in H_1, then there exists a time schedule $S = [0, \tau > 0]$ during which every flow g is active during a period τ_g and flows τx_g bits. Since H_1 is complete, then $\tau_f \cap \tau_h = \emptyset$ for all $f \neq h$. This implies that $\sum_{g \in \mathcal{F}_1} \tau_g = \tau$. Now since $\mathbf{x}$ is feasible, for every $g \in \mathcal{F}_1$, $\tau x_g \leq \tau_g C$. By summing over all flows, we

obtain $\tau \sum_{g \in \mathcal{F}_1} x_g \leq \sum_{g \in \mathcal{F}_1} \tau_g C = \tau C$. By simplifying by τ, it follows that $\sum_{g \in \mathcal{F}_1} x_g \leq C$. Now, since $\sum_{g \in \mathcal{F}_1} x_g = \delta(\Psi_f, \mathbf{x}) + x_f$, then $x_f \leq C - \delta(\Psi_f, \mathbf{x})$.
1) : From 3), it follows that $x_f \leq C - \delta(\Psi_f, \mathbf{x})$; which implies that $x_f \leq \max\{\frac{C}{d_f+1}, C - \delta(\Psi_f, \mathbf{x})\}$.
2) : We proceed with a counter example. Consider the flow contention complete graph $H = (\{f, g\}, \{(f, g)\})$ (H consists of only two contending flows f and g). Let $\mathbf{x} = (1, 0)$ be a flow rate vector in H (assume C equals 1). Note that $\mathbf{x}$ is feasible, but it does not pass `DegreeFT` since $x_f = 1 > \frac{1}{2} = \frac{1}{d_f+1}$.■

For some MAC protocols, the resulting flow contention graphs are unions of complete graphs. Examples of networks using such MACs are Bluetooth and sensor networks. For these MACs, we can apply Theorem 5 to determine the necessary and sufficient MAC constraints. We illustrate this in the following example.

Example 5 *In this example, we consider the WINS (Wireless Integrated Network Sensors) NG 2.0 platform-based network [7, 13]. The WINS node has the ability to communicate simultaneously on two independent Modems. Each Modem can be configured independently as either* remote *or* base*. A remote Modem must be associated with a base Modem of another node. Each node can be configured so that it can have two concurrent flows. Remotes that communicate with the same base constitute a cluster. Neighboring clusters use different radios so that they do not interfere with each other. A cluster can have at most 8 remotes. Within a cluster only one communication can occur at a time. Communication across clusters is allowed only through base nodes. We consider the WINS network given in Fig. 6(a). Let $H = (\mathcal{F}, \mathcal{C})$ be the flow contention graph of the WINS network. Note that H is a union of three complete graphs. Since all flows in cluster 1 contend for accessing the medium of the base of node n_3, and no other flows in the network contend for that medium, the corresponding flow contention graph of these flows constitutes one disjoint complete graph in H. Similarly, the flow contention graphs of all flows in cluster 2 and cluster 3 constitute the other two disjoint complete graphs in H. Fig. 6(b) shows these three disjoint complete graphs forming the union H. Therefore, Theorem 5 can be used to impose necessary and sufficient MAC constraints.*

In fact, for this MAC protocol, it can be shown that the flow contention graph is always a union of one or more complete graphs. As a result, one can use always the result of this paper (Theorem 5) to impose necessary and sufficient MAC constraints. We have applied this to solve the data aggregation problem in the SensIT testbed [6]. ■



(a) Network graph G.

(b) Flow contention graph H.

Figure 6: Graphs for Example 5.

IV. Performance study

To evaluate and compare the performance of the three feasibility tests given in Section III, we conduct two types of simulation studies. First, we evaluate the effectiveness of these tests by considering flow contention graphs as opposed to network graphs to mask the effects of MAC protocols. Second, we evaluate the admission control capability of each of the three tests by simulating IEEE 802.11 MAC-based wireless ad hoc networks.

IV.A. MAC-independent evaluation

We generated and simulated random flow contention graphs, each consisting of 100 flows. In each graph, two flows contend with each other with probability 0.02. That is, the probability that flow i contends with flow j is 0.02 for all i and j, and hence, on average, each flow contends with 2 flows. The capacity of the wireless medium is assumed to be unity. Rate components of the flow rate vector $\mathbf{x}$ associated with each graph are selected from a uniform distribution in the range $[\bar{R} - \Delta R, \bar{R} + \Delta R]$. The average $\bar{R}$ of the rates is set to 0.07. The *coefficient of variation* C_R is var-

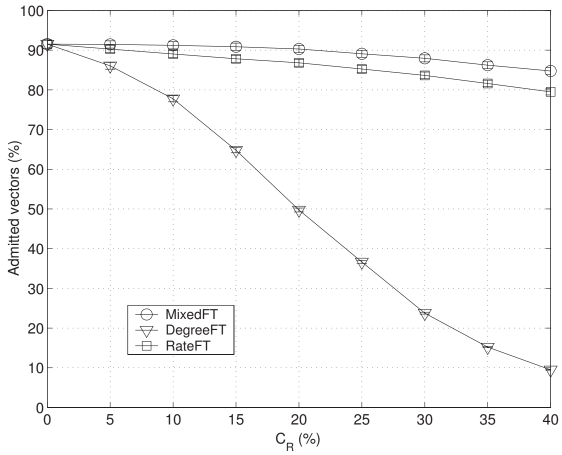


Figure 7: Comparison of `MixedFT`, `DegreeFT` and `RateFT`—confidence level $= 98\%$, confidence interval $= 1\%$.

ied between 0% and 40%, and thus ΔR varies from 0 to 0.0485 ($\Delta R = \sqrt{3}\bar{R}C_R$). For every hundred combinations—a combination consists of a random graph and a random flow rate vector, we collect the percentage of *admitted vectors* under the three feasibility tests: `MixedFT`, `DegreeFT`, and `RateFT`. We repeat simulations until the measured metric (i.e., percentage of admitted vectors) converges to within 1% of the real value at a confidence level of 98%.

The percentage of admitted vectors as a function of the coefficient of variation is shown in Fig. 7 for the three feasibility tests. First, note that as expected from the results in Section III, `MixedFT` always admits more rate vectors than `DegreeFT` and `RateFT`. Second, observe that while the percentage of admitted vectors is the same for each test when rates have no variation, it decreases as the coefficient of variation increases. The higher the coefficient of variation, the more unbalanced the contention, which in turn results in less feasible rate vectors.

It is worth mentioning that since `MixedFT` always does better than the other two tests–even in the presence of rate variability, one can suggest to use always `MixedFT` instead of `DegreeFT` or `RateFT`. Unfortunately, `MixedFT` is not well suited for all applications. For instance, `MixedFT` cannot be used in the LPP routing formulation discussed in Example 4 since the constraints are not linear. `DegreeFT` or `RateFT`, however, can be used to include MAC contention constraints in this LPP formulation. Because `DegreeFT` and `RateFT` are simpler in their formulation than `MixedFT`, we compare their performance to `MixedFT`.

Fig. 7 shows that independently of the coefficient of variation, `RateFT` always results in more admitted vectors than `DegreeFT`. Also, note that the higher the coefficient of variation, the larger the gap difference between the two achieved percentages of admitted vectors. This gap reaches approximately 70% when the coefficient of variation achieves 40%. Moreover, observe that the gap difference between the percentage of admitted vectors under `RateFT` and that under `MixedFT` is about 6% for rate vectors with high variability; whereas the gap reaches 75% in the case of `DegreeFT`. In other words, among all simulated vectors, 6% of those that pass `MixedFT` fail `RateFT`; whereas 75% of the vectors that pass `MixedFT` fail `DegreeFT`. For completeness, we also measured the percentage of admitted vectors that satisfy the conditions of `DegreeFT`, but not those of `RateFT`. This percentage (not shown in the figure) achieves about 1% at high rate variability. That is, only about 1% of the vectors that pass `DegreeFT` fail `RateFT`.

By looking at Theorems 1 and 2, we note that the upper bound on the rates stated by `DegreeFT` depends only on the flow contention graph and not on the rates themselves. In the case of `RateFT`, however, the upper bound depends on both the flow contention graph and the rates. Therefore, in applying `RateFT`, lowering the rates of some flows would allow others to have their rates increased without violating the upper bound constraints. This is not the case for `DegreeFT`. Hence, flow rate vectors with high variation are more likely to pass `RateFT` than `DegreeFT`. This explains the behaviors observed in the previous paragraph.

Trade offs in using these feasibility tests exist depending on where to apply them. While `DegreeFT` results in a worse performance than `RateFT` in the case of rates with high variability, `DegreeFT` is easy to implement and verify. For example, if one needs to apply `DegreeFT` in an admission control routing mechanism for wireless networks, a node only needs to know the topology in order to admit a flow. But, if `RateFT` is applied instead, then to admit a flow, a node not only needs to know the topology, but also all the rates of flows that contend with the concerned flow.

In summary, `DegreeFT` fits better in applications where flows tend to have the same rates. Conversely, when applications present rates that may substantially vary from one node to anther, `RateFT` applies better than `DegreeFT`. In all these applications, however, `MixedFT` still performs better than the other two feasibility tests.

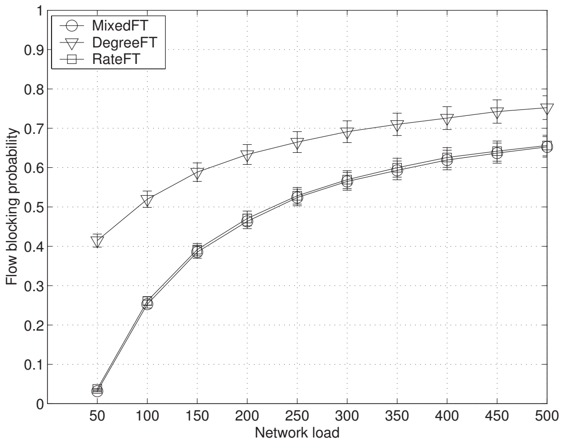


Figure 8: Admission control comparison of `MixedFT`, `DegreeFT`, and `RateFT`—$C_R = 40\%$, confidence level $= 98\%$, confidence interval $= 5\%$.

IV.B. IEEE 802.11 MAC-based evaluation

In this section, we study the admission control capability of the three feasibility tests in IEEE 802.11 MAC-based networks. To mimic IEEE 802.11 MAC-based networks, we simulate random wireless ad hoc networks each of which consists of 50 nodes. Nodes are uniformally distributed in a cell of size 100×100 meters square where two nodes are considered neighbors if the distance between them does not exceed 14 meters. During the course of simulations, end-to-end flows are generated randomly according to a Poisson process of arrival rate λ. Each end-to-end flow is characterized by (i) a random pair (source-destination) of nodes, (ii) a chain of one-hop flows constituting the shortest path between these two nodes, (iii) a flow rate selected from a uniform distribution in the range of $[\bar{R} - \Delta R, \bar{R} + \Delta R]$ as defined in Subsection IV.A, and (iv) an exponentially distributed duration of rate μ. We set $\bar{R}$ to 0.01 and simulate for $C_R = 40\%$. We define $\mathcal{L} = \frac{\lambda}{\mu}$ to be the *network load*. The simulator collects the blocking probability of end-to-end flows for each of the three feasibility tests. When a new end-to-end flow arrives, the flow is admitted to the network only if all flows (already existing flows and the new flow) satisfy the sufficient conditions of the corresponding test. Simulations continue until the measured *flow blocking probability* converges to within 5% of the real value at a confidence level of 98%.

Fig. 8 shows the flow blocking probability of each test as a function of network load $\mathcal{L}$ when $C_R = 40\%$. As expected, note that the blocking probability increases as the network load increases. This probability varies depending on both the load and the test. Observe that `DegreeFT` achieves the worst performance among all the three tests; whereas `RateFT` and `MixedFT` achieve comparable performance.

V. Conclusion

In this paper, we prove sufficient conditions (referred to as feasibility tests) under which flow rate vectors are feasible given a MAC protocol. We show that these tests apply to a wide range of MAC protocols including IEEE 802.11 and TDMA. We show that the tests can be applied and are useful in solving many problems such as linear programming-based routing problems and admission control mechanisms for QoS routing. We also prove that some of the sufficient conditions are also necessary for certain MAC protocols such as those used by the WINS sensor [7] and the Bluetooth [11] networks. Through illustrative examples, we show the importance of including the physical contention constraints by means of the proposed feasibility tests in determining routing solutions and admitting new flows into the network. We evaluate and compare the performance of the proposed tests through simulation studies. Based on the obtained numerical results, we discuss the applications of these conditions.

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