# Optimal Sensor Distribution for Maximum Exposure in A Region with Obstacles

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***Abstract*— Sensor networks have been envisioned to enhance the ability of human beings in observing the environment and understanding the world. A potential application of a sensor network is to detect the presence or absence of a target in a region of interest. Many heuristics have been proposed in literature for placing sensors to achieve better coverage in the monitored region. However, none of them guarantee an optimal sensor deployment especially when there are obstacles in the region. Unlike the prior work, this paper focuses on the problem of determining the optimal sensor distribution in a region with or without obstacles. The detection performance is characterized using a metric called "exposure", which is defined as the least probability of detecting a target over all possible target locations subject to a fixed false alarm probability. A linear programming based approach is proposed to find the optimal sensor distribution by maximizing the exposure in a given region with or without obstacles. The optimal sensor distribution can also be used as weights of sensor measurements taken at different locations for decision-making.**

## I. Introduction

Advances in integrated circuit design coupled with improvements in wireless network technology have enabled the emergence of low-cost low-power devices capable of measuring the environment, exchanging information, and processing queries. Such devices can form a so called "sensor network" through wireless communication to monitor a region of interest. With the advantages of easy deployment, efficient data collection, and flexible network structure, sensor networks are able to support the missing connection between computational systems and the physical environment. Numerous applications which were difficult, if not impossible, in the past have now been realized by sensor networks [1]. The applications range from battlefield surveillance [2], [3], to ecological observation [4], [5] to environmental monitoring [6], [7]. This paper addresses the fundamental problem of deploying sensors to well cover the region of interest. Many heuristics have been proposed in literature in order to determine better sensor locations [8]–[11]. However, none of them guarantee that the final deployment will reach the optimal especially when there are obstacles in the region. In contrast, this paper proposes a linear programming based approach to determine the optimal sensor distribution in a region with or without the presence of obstacles.

To detect the presence of a target, a collaborative sensing method is used to make consensus decisions. Following the method, sensors take local measurements of the target signal and report their measurements to a fusion center periodically. In each period, the measurements from many/all sensors are fused to arrive at a consensus decision about the presence or absence of the target. Obstacles in the region may distort the target signal and reduce the detection probability, and noise in the environment may corrupt the sensor measurements and cause false alarm. In general, the detection probability is highly correlated with the false alarm probability. In this paper, the performance of a sensor network is measured using a metric called "exposure", which is defined as the *least* probability of detecting a target over all possible target locations in the region subject to a fixed false alarm probability. The higher the exposure, the better the deployment.

A linear programming based approach is proposed to find the optimal sensor distribution to maximize the exposure. In particular, the linear programming formulation is applicable to both sensor networks with and without the presence of obstacles. The paper demonstrates the optimal sensor distributions for both cases. The optimal sensor distribution can also be used as the weights of sensor measurements taken at different locations. The weights are useful for consensus decision-making if the locations, where the measurements are taken, are known. The result brings the capability of each sensor into full play and is desired in many applications especially when the target is malicious or hostile.

The paper is organized as follows. Section II reviews the related work in recent literature. Section III describes the target model and exposure evaluation method. Section IV presents the linear program approach for determining the optimal sensor distribution. Section V demonstrates the results of the optimal sensor distribution in regions with and without obstacles. Section VI discusses different usages of the proposed approach. The paper concludes in Section VII.

## II. Related work

Exposure and coverage of sensor networks have been studied extensively in literature [12]–[16]. In [12], exposure is defined as the integral of signal intensity measured by sensors when a target moves across a region. The effectiveness of a deployment is characterized as the minimum exposure over all possible target paths in the region. By converting the target paths to a high order grid in the region, a method based on a shortest path algorithm is proposed to find the minimum exposure path. Localized algorithms for calculating such intensity-based exposure are proposed in [13] and [17]. Although signal intensity is an indicator of the likelihood of detecting a target, the relationship between signal intensity and detection probability is not linear. In [3], the authors directly work on detection probability to determine the exposure. Exposure is defined as the minimum probability of detecting a target over all possible target maneuvers in the region. The proposed method divides the region into a grid and associates each grid edge with a weight according to the detection probability of a target moving on the grid edge. It then finds the exposure path using the shortest path algorithm. Networked mobile sensors are also being used for target detection in surveillance networks [18], [19]. In such networks, mobile sensors take measurements and make consensus decisions while moving around the region. The exposure path associated with a time for the target to enter the region and a path for it to traverse in the region is studied in [18];

Considerable efforts have been made for strategies of deploying sensors. In [11], a method is proposed to deploy a limited number of nodes at a time until a desired minimum exposure is achieved. The tradeoff between exposure and the cost of sensors and deploying processes is studied for choosing the number of nodes deployed at each step. Recent research has focused on methods of using node mobility to assist deployment and improve the coverage in sensor networks [8]–[10]. In [8], a method based on the concept of potential field is proposed. From an initial random deployment, sensors are repelled to spread out by a virtual potential field produced by sensors and/or obstacles. A similar method based on virtual forces is proposed in [9]. Attractive and repulsive virtual forces are generated between nodes, obstacles, and spots of interest according to the distance between them. The algorithm iteratively moves sensors to balance the virtual forces until a threshold of coverage is reached. In [10], the authors use Voronoi diagrams to identify coverage holes in a network and move sensors to improve the coverage.

Unlike the prior work which only evaluates the exposure or provides heuristics for sensor deployments, this paper determines the optimal sensor distribution for a given region with or without obstacles. The paper adopts the probability-based exposure for measuring the quality of a deployment and finds the optimal sensor distribution in the region. Moreover, since sensor measurements are inherently corrupted by noise, decisions made by an individual sensor are usually not reliable. This paper investigates sensor networks which use a collaborative sensing method. The collaborative sensing method provides more robust results and improves the detection performance.

## III. EXPOSURE EVALUATION

### A. Target model

Consider a target at a certain location in a region of interest. Based on the general radio propagation model [20], the signal energy of the target is assumed to decay as a power of the distance from the target. Let $K$ denote the signal energy emitted by the target and $\beta$ be the decay factor of the energy.

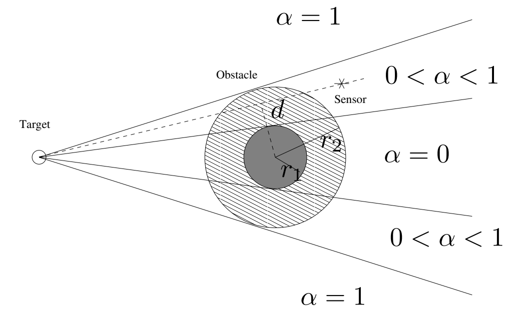


Fig. 1. The model of distortion caused by an obstacle.

If the target is at location $u$, the target energy measured by a sensor at location $s$ is modeled as

$$S(u) = \begin{cases} \frac{\alpha(u)K}{|u-s|^\beta} & \text{if } |u-s| > 1 \\ K & \text{otherwise} \end{cases}, \tag{1}$$

where $|u - s|$ is the Euclidean distance between the target and the sensor, and $\alpha(u)$ is the energy distortion factor due to obstacles. Depending on the environment, the decay factor $\beta$ is typically between 2 to 5 [21].

The energy distortion factor $\alpha(u)$ characterizes the impact of obstacles on the energy detected by sensors. Fig. 1 shows a model for the energy distortion factor. In this model, an obstacle is characterized using two radii, $r_1$ and $r_2$, with $r_1 \leq r_2$. Given a target, an obstacle, and a sensor, the obstruction distance $d$ is defined as the perpendicular distance from the center of the obstacle to the line joining the target and the sensor. If a sensor lies between the two tangent lines from the target to the inner circle, $\alpha(u) = 0$. If a sensor lies outside the two tangent lines from the target to the outer circle, $\alpha(u) = 1$. Between the inner and outer tangent lines, $\alpha(u)$ increases linearly from zero to one with the distance $|d - r_1|$. If there are multiple obstacles between the target and the sensor, the distortion factor is the product of the distortion caused by each obstacle. Obstacles are assumed to be impenetrable which means the target and sensors cannot reside in the area occupied by obstacles.

The model shown in Fig. 1 is a simple approximation illustrating energy absorbed by a round obstacle. With slight modification, the model can also be applied to obstacles of other shapes and characteristics. For example, the effect of energy reflected by obstacles can be considered by choosing appropriate values for $\alpha(u)$.

Since sensor measurements are usually corrupted by noise, the total energy measured by sensor $i$ is modeled as the sum of target energy and noise energy, i.e.,

$$E_i(u) = S_i(u) + N_i^2,$$

where $N_i^2$ denotes the noise energy detected by sensor $i$.

### B. Detection

Consider that $n$ sensors are deployed in the monitored region. For detecting the target, collaborative sensing method proposed in [3] is adopted to reduce false alarm. In the method, sensors take measurements periodically and report the energy readings to a fusion center. The fusion center compares the sum of the energy readings to a threshold $\eta_n$. If the sum is greater than $\eta_n$, it decides that a target is present. Otherwise, it decides that there is no target. Based on the collaborative sensing method, the probability of detecting a target at location $u$ is given by

$$D(u) = Prob\left(\sum_{i=1}^{n}\left(S_i(u) + N_i^2\right) > \eta_n\right) \quad (2)$$

$$= Prob\left(\sum_{i=1}^{n} N_i^2 > \eta_n - \sum_{i=1}^{n} S_i(u)\right), \quad (3)$$

where $\eta_n$ is a energy threshold chosen by a fixed false alarm probability. If noise process at each sensor is independent, the probability density function of $\sum_{i=1}^{n} N_i^2$ is the convolution of the probability density functions of $N_i^2, i = 1, ..., n$. In particular, if $N_i$ is Additive White Gaussian Noise (AWGN) with mean zero and variance one, then $N_i^2$ is Chi-square with one degree of freedom and $\sum_{i=1}^{n} N_i^2$ is Chi-square with $n$ degrees of freedom.

Due to noise, the consensus decision may incorrectly detect a target when there is actually no target in the region, i.e., $S_i(u) = 0$. The probability of false alarm is given by

$$F_a = Prob\left(\sum_{i=1}^{n} N_i^2 > \eta_n\right).$$

In particular, if the noise process is AWGN with mean zero and variance one, the false alarm probability can be obtained by Chi-square distribution with $n$ degrees of freedom. Given a tolerable false alarm probability $F_a$, one can determine the corresponding fusion threshold $\eta_n$. In general, detection probability and false alarm probability are closely related. A higher detection probability always comes with a higher false alarm probability.

### C. Exposure

Probability-based exposure is used for evaluating the quality of a sensor deployment in this paper. Given a fixed number of sensors deployed in a region, the higher the exposure, the better the sensor network. To facilitate computation, the region is divided into a grid $G_t$ for the locations of the target. The granularity of the grid trades off the computational load with accuracy. The set of target locations is defined as follows.

**Definition 1.** ***Target locations*****:** Let $O$ denote the area occupied by the obstacles. Assume that the target is equally likely to appear on the grid points which are not occupied by obstacles in $G_t$. The set of target locations is defined as $T = \{t_i | t_i \in G_t \text{ and } t_i \notin O\}$.

Let $\mathcal{E}$ denote the exposure of a target. The exposure is formally defined as follows.

**Definition 2.** ***Exposure*****:** Exposure is defined as the least probability of detecting a target over all possible target positions, i.e.,

$$\mathcal{E} = \min_{u \in T} D(u). \quad (4)$$

where $D(u)$ is the probability of detecting a target at location $u$ as in Eq. (2).

## IV. Optimal sensor distribution

### A. Problem

Exposure has been used as a metric to evaluate the quality of a sensor network. A fundamental problem for a sensor network is to deploy the sensors such that the region is better covered in terms of exposure. In other words, the optimal sensor deployment is desired to be known before deploying the sensors. The optimal sensor deployment is extremely important for surveillance systems especially when the target is malicious or hostile. In practice, if there are obstacles such as trees and buildings in the region, an intelligent target can hide in the "shadow" of the obstacles to reduce the probability of being detected. Clearly, the size and locations of obstacles will also affect the optimal sensor distribution. Our goal is to find the optimal sensor distribution in terms of exposure in a given region with or without the presence of obstacles. The optimal sensor distribution provides a basis for deploying sensors in the monitored region.

### B. Solution

From the definition, exposure is the least probability of detecting a target over all possible target locations. To maximize exposure is equivalent to maximizing the minimum detection probability, which can be achieved by maximizing the minimum sum of detected target energy over all possible target locations from Eq. (2). Thus, instead of dealing with exposure directly, we focus on the sum of energy detected by sensors over all target locations.

Let us consider a square region with the presence of obstacles. To approximate the continuous sensor density in a discrete fashion, the region is divided into a grid $G_r$. Each small square in the grid is called a cell. Let $\rho(\cdot)$ denote the sensor mass function sampled at the center of each cell over the region. The number of sensors residing in cell $k$ is given by $\rho(r_k) \cdot A$, where $A$ is the area of a cell and $r_k$ is the center of the cell $k$. Without loss of the generality, the center of each cell in $G_r$ is chosen as the location of the sensors in the cell. To calculate exposure, the set of sensor positions is defined as follows.

**Definition 3.** ***Sensor positions*****:** Let $C_r$ be the set of cell center locations in $G_r$. Assume that a cell is not occupied by obstacles if the center of the cell is not inside any obstacle. The set of sensor positions is defined as $R = \{r_k | r_k \in C_r \text{ and } r_k \notin O\}$.

It is noted that the grid $G_t$ for target locations can be different from $G_r$ in granularity. Let $S_k^i$ be the target energy detected by a sensor at $r_k \in R$ when the target is at $t_i \in T$. The objective of the problem is to maximize the minimum sum of target energy detected by sensors over all possible target positions. Thus, the objective can be formulated as the

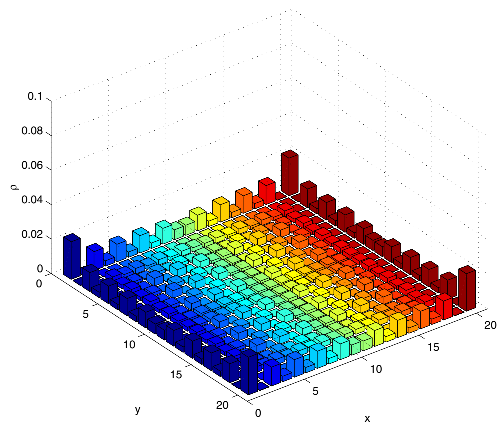


Fig. 2. The optimal sensor distribution in the absence obstacles.

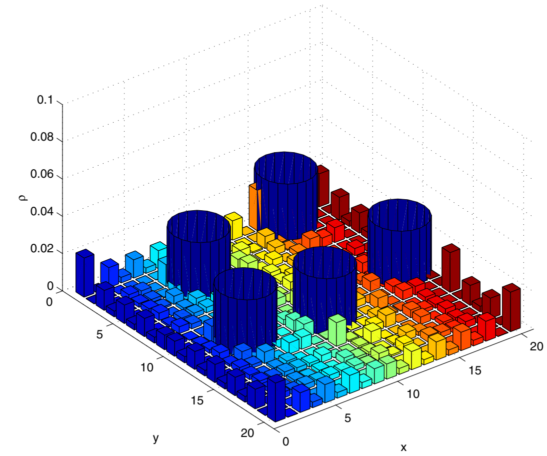


Fig. 3. The optimal sensor distribution for a region with obstacles.

following linear programming problem.

$$\max\ \epsilon \tag{5}$$

$$s.t. \tag{6}$$

$$\sum_{r_k \in R} \rho(r_k) A S_k^i \geq \epsilon,\ \forall\, t_i \in T \tag{7}$$

$$\sum_{r_k \in R} \rho(r_k) A = 1 \tag{8}$$

$$\rho(r_k) \geq 0,\ \forall\, r_k \in R, \tag{9}$$

where $\epsilon$ is the minimum sum of target energy detected by the sensors. Eq. (7) describes that the sum of the detected target energy for each possible target position is no less than $\epsilon$. Eq. (8) and Eq. (9) represent that the sum of a probability mass function is one and the value of the mass probability is non-negative. Clearly, the solution of the function $\rho(r_k)$ in this problem is the optimal sensor distribution over the region. Note that the energy distortion caused by obstacles is implied in $S_k^i$, which is calculated using Eq. (1). The presence of obstacles makes the problem more complicated but does not change the formulation of the approach to find the optimal sensor distribution.

## V. Demonstration

### A. No obstacles

The optimal sensor distribution in a region without obstacles is first calculated in a $20 \times 20$ region. The region is divided into grids for both target positions and sensor positions, i.e. $G_t$ and $G_r$ respectively. For simplicity of presentation, the grid size for both grids is chosen to be one. The target may appear on any grid point. The energy emitted by the target is $K = 30$ and the decay coefficient is $\beta = 2$. The fusion threshold $\eta_n$ is chosen such that false alarm probability is 0.001. These attributes used in the experiments can be replaced according to the characteristics of the target.

Applying the linear programming based approach, the optimal sensor distribution in the region without obstacles is shown in Fig. 2. A general pattern of the optimal sensor distribution is exhibited in the figure. Sensors have a higher probability of being located in the border than in the interior of the region. The boundary effect compensates for no sensor deployed outside the region. The high density prevents the border from becoming the weakest spot of the region. The sensor distribution has very regular pattern in the interior of the region since the target is equally likely to be present on any grid point. If the target can appear on any location in the continuous region, a reasonable sensor distribution in the interior of the region should be uniform.

Intuitively, the result follows the assumption that the target can appear at any target location with equal probability. Consequently, all the possible target locations must be equally covered and the detection probability is the same at any location in the region. If the target has higher probability to appear at certain locations, for example, area along roads or around important assets, the optimal sensor distribution should exhibit higher density at these locations.

### B. Obstacles

We also demonstrate the case with the presence of obstacles in the region. The attributes used in the experiment are the same as those in the previous one except five obstacles are randomly placed in the region. The two radii characterizing the obstacles are assumed to be $r_1 = 1$ and $r_2 = 2$. The optimal sensor distribution for an example terrain is shown in Fig. 3. The boundary effect is also seen in the result, but the sensor distribution is irregular in the interior. Higher density is required around the obstacles such that the "shadow" created by the obstacles is lightened. The density highly depends on the locations of obstacles in the region. Unlike the previous experiment, the optimal sensor distribution in a region with obstacles is hard to precisely predict even when the target appears on all target locations with equal probability.

## VI. Discussion

The linear programming based approach provides the optimal sensor distribution in the monitored region. Alternatively, the result can be viewed as revealing the importance level for measurements taken at different sensor locations. In other words, if one sensor is deployed in each sensor location, the result of the linear programming based approach provides a weight for the piece of information reported from a particular sensor location. Therefore, to maximize the exposure can also be realized by deploying a sensor at each sensor location and fusing the weighted measurements to arrive at consensus decisions.

In some applications, sensors are not deployed at regular locations in a monitored region. For example, sensors air-dropped from an aircraft render a random deployment. The weighted fusion can also be used in such deployments if sensor locations can be determined using Global Positioning System (GPS) or other localization algorithms. The optimal weight for each sensor location can be calculated by the proposed linear programming based approach. Detection performance can be substantially increased if the optimal weighted measurements are fused to make decisions.

Further, if each cell in $G_r$ is allowed to hold at most one sensor, it is possible to change the linear programming problem to an integer programming problem in order to find a set of best sensor locations. The integer programming problem is formulated as follows.

$$\max\ \epsilon \tag{10}$$
$$s.t. \tag{11}$$
$$\sum_{r_k \in R} \rho(r_k) S_k^i \quad \geq \quad \epsilon, \quad \forall\, t_i \in T \tag{12}$$
$$\sum_{r_k \in R} \rho(r_k) \quad = \quad n \tag{13}$$
$$\rho(r_k) \quad = \quad 0 \text{ or } 1, \quad \forall\, r_k \in R \tag{14}$$

Eq. (13) provides the constraint on the total number of sensors and $\rho(r_k)$ can only be zero or one as shown in Eq. (14). Indeed, since solving integer programming problem is time consuming, this method is appropriate for networks with a small number of sensors. However, deploying sensors in the best locations to effectively perform detection operations is always desirable.

## VII. Conclusion

This paper addresses the problem of finding the optimal sensor distribution in a region with or without obstacles. A linear programming based approach is proposed to determine the optimal sensor distribution. The approach maximizes the minimum sum of target energy detected by the sensors over all possible target positions. From the results, we found the optimal sensor distribution for a region without obstacles is intuitively justifiable for the higher density on the boundary and regular distribution in the interior of the region. However, the optimal sensor distribution is considerably different in the presence of obstacles.

This work focuses on detecting an idling target. It is sufficient for some cases such as detecting a bomb or a heat source. Nevertheless, many applications deal with traversing targets such as vehicles or animals. The optimal sensor distribution for a traversing target can be quite different from that for an idling target. To determine the optimal sensor distribution and design effective deployment strategies for different target activities are open for further investigation.

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