Rate Feasibility under Medium Access Contention Constraints

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***Abstract*— Wireless nodes within the same vicinity contend for accessing the shared medium. The contention constraints on sharing the medium depend on the medium access control (MAC) protocol. For example, in IEEE 802.11 MAC protocol-based networks, if node i is in communication with node j, then all nodes within the same transmission range of i or j cannot communicate. On the other hand, if nodes within each other's transmission range can use different frequencies (e.g., FDMA) or different codes (e.g., CDMA), then neighbor nodes can communicate simultaneously. Furthermore, if nodes are equipped with two radios (e.g., WINS sensor [1] networks), then nodes not only can communicate concurrently but also can receive while they are transmitting. In this paper, we prove a sufficient condition under which a flow rate vector is feasible given the MAC protocol. We also prove that the sufficient condition is necessary for some MAC protocols such as those used by WINS sensor [1] and Bluetooth [2] networks. We give illustrative and real examples for which these conditions apply.**

***Index Terms*— Medium access control (MAC) protocol, wireless networks, sensor networks, flow rate feasibility, MAC contention constraints.**

I. INTRODUCTION

In wireless networks (e.g., ad hoc, sensor and Bluetooth networks), nodes within the same vicinity contend for the wireless shared medium. For example, in IEEE 802.11 medium access control (MAC) protocol-based networks, if a node i is communicating with a neighboring node j, then all nodes within the communication range of node i or j must remain idle. On the other hand, if nodes within the same transmission range use different frequencies (e.g., FDMA) or different spreading codes (e.g., CDMA), then two neighboring nodes can simultaneously use the wireless medium. Flows in these networks which do not share nodes can communicate at the same time. Furthermore, in networks where nodes are equipped with two radios (e.g., WINS networks [1, 3]), a node can be simultaneously transmitting and receiving. Therefore, we can conclude that the achievable flow rates of a given network depend on the MAC protocol and the capabilities of the nodes constituting the network.

Medium contention constraints have been used for different objectives [2, 4–8]. In [4], wired link contention is used as a flow constraint to find the system optimal rates. In [5, 6], MAC contention constraints are used as a means of guaranteeing the feasibility of schedules. In [4–6], the only constraint on the medium accessibility is that a node can only transmit or receive at a time. They assume systems consist of nodes that, for example, use different frequencies or different codes if they are within each other's vicinity.

Some linear programming-based routing solutions for wireless networks (e.g., sensor and Bluetooth networks) use the required flow rates to express the flow conservation constraints [2, 7, 8]. However, medium access constraints are not imposed in solving these problems to ensure the feasibility of the required rates (e.g., in [7, 8]). If MAC constraints are not considered in solving such routing optimization problems, then the resulting rate solutions may be infeasible in the sense that the MAC protocol cannot satisfy the required rates.

This paper proves a sufficient condition under which a flow rate vector is feasible given the MAC protocol. We prove that the sufficient condition is also necessary for some MAC protocols such as those used by the WINS sensor [1] and the Bluetooth [2] networks. Through illustrative examples, we show the importance of including the physical contention constraints in determining routing solutions.

The paper is organized as follows. Section II presents the network model. In Section III, we prove sufficient and necessary conditions for rate feasibility. We present illustrative examples in Section IV. Finally, we conclude the paper in Section V.

II. NETWORK MODEL

We consider a wireless network of nodes. Let N be the set of nodes. An ordered pair of nodes $(i, j) \in N^2$ is said to form a flow f if i directly communicates with j. The flow f is said to be *active* if i is currently communicating with j otherwise the flow is said to be *inactive*. If two flows f and g cannot be active at the same time, then we say that f and g contend with each other. Let F be the set of all the flows in the network. We model the wireless network as a directed graph $\mathcal{G} = (N, F)$. We assume that $\mathcal{G}$ is simple [1]; which means that (i) if a pair of nodes $(i, j) \in N^2$ forms a flow in F, then nodes i and j

[1] We use the same definition as in [9].

cannot be the same (i.e., loop-free), and (ii) each ordered pair of nodes can form at most one flow.

We define the *contention set* Φ_f of a flow f to be the set consisting of the flow f and all flows in F contending with f. Formally, $\Phi_f = \{f\} \cup \{g \in F : f$ and g contend with each other$\}$. Let $\bar{\Phi}_f$ denote $F - \Phi_f$. It is worth noting that two flows belonging to the same contention set can be simultaneously active. However, no flow $g \in \Phi_f$ may be simultaneously active with f. Given a MAC protocol, we model the flows and their contention in the network $\mathcal{G}$ as a contention flow graph $\mathcal{H} = (F, E)$ where again F is the set of flows, and each edge $e \in E$ corresponds to a contending pair of flows in F.

Let us assume that each flow f in F flows data traffic at a rate of x_f bits per second. We call *flow rate vector* the vector $x = (x_g)_{1 \leq g \leq |F|}$ denoting the data rates of all flows in F ($|F|$ denotes the cardinality of the set F). We define the *weight* $\delta(f, x)$ of a flow f in F under a flow rate vector x to be $\sum_{g \in \Phi_f} x_g$. The flow rate vector $x = (x_g)_{1 \leq g \leq |F|}$ is said to be *feasible* in $\mathcal{H}$ if there exists a time schedule during which all the rate components x_g, $1 \leq g \leq |F|$, are satisfied. Formally, x is feasible in $\mathcal{H}$ if there exists a time schedule $S = [0, \tau]$ of length $\tau > 0$ during which every flow $f \in F$ communicates τx_f bits. Let C denote the capacity of the wireless medium.

III. Conditions for Rate Feasibility

In this section, we first prove a sufficient condition under which a given flow rate vector is feasible subject to certain MAC contention constraints. Then, we prove that if the network is complete [2], the sufficient condition is also necessary.

Lemma *1:* Let $x = (x_g)_{1 \leq g \leq |F|}$ be a flow rate vector and f be the flow in F with the largest weight; i.e., $\delta(f, x) \geq \delta(g, x), \forall\, g \in F$. Then, for all $h \in \bar{\Phi}_f$,

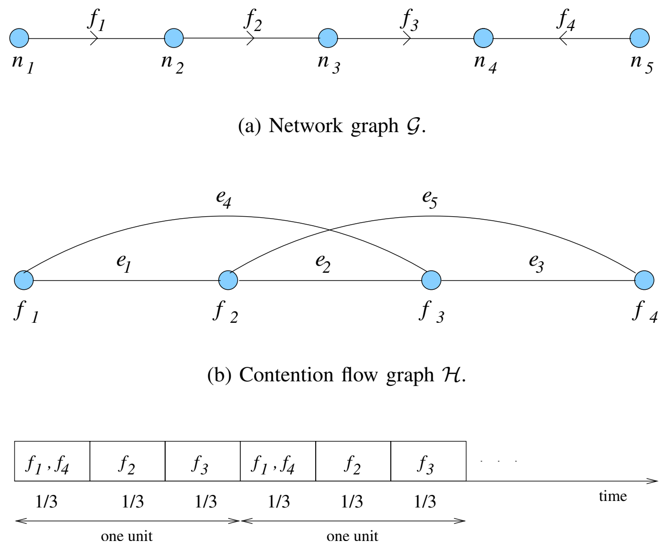
$$\sum_{g \in \Phi_f - \Phi_h} x_g \geq \sum_{g \in \Phi_h - \Phi_f} x_g.$$

Proof of Lemma 1: Recall that $\bar{\Phi}_f = F - \Phi_f$. From hypothesis, we have $\sum_{g \in \Phi_f} x_g \geq \sum_{g \in \Phi_h} x_g$ (since $\delta(f, x) \geq \delta(h, x)$). By writing $\Phi_f = \{\Phi_f - \Phi_h\} \cup \{\Phi_f \cap \Phi_h\}$ and $\Phi_h = \{\Phi_h - \Phi_f\} \cup \{\Phi_f \cap \Phi_h\}$, it follows that $\sum_{g \in \Phi_f - \Phi_h} x_g + \sum_{g \in \Phi_f \cap \Phi_h} x_g \geq \sum_{g \in \Phi_h - \Phi_f} x_g + \sum_{g \in \Phi_f \cap \Phi_h} x_g$. Simplifying the term $\sum_{g \in \Phi_f \cap \Phi_h} x_g$ from both sides, we obtain $\sum_{g \in \Phi_f - \Phi_h} x_g \geq \sum_{g \in \Phi_h - \Phi_f} x_g$.■

Proposition *1:* Let $x = (x_g)_{1 \leq g \leq |F|}$ be a flow rate vector. The vector x is feasible in $\mathcal{H}$ if for all $h \in F$, $\delta(h, x) \leq C$.

Proof of Proposition 1: Let f be the flow in F with the largest weight; i.e., $\delta(f, x) \geq \delta(g, x), \forall\, g \in F$. We need to show that there exists a time schedule S of length τ during which each flow $h \in F$ flows at least τx_h bits. Consider the time schedule $S = [0, \tau]$ where the length τ equals $\sum_{g \in \Phi_f} x_g$ seconds. We proceed the proof in two steps. In the first step, we schedule flows in Φ_f, and in the second step, we schedule flows in $\bar{\Phi}_f$.
Step 1: Let each flow $h \in \Phi_f$ be active during x_h seconds, and all flows in $\bar{\Phi}_f$ be inactive all time. Since $\sum_{g \in \Phi_f} x_g \leq C$, then, by multiplying both sides by x_h, it follows that $x_h \sum_{g \in \Phi_f} x_g \leq x_h C$ which translates to $x_h \tau \leq x_h C$ ($\sum_{g \in \Phi_f} x_g = \tau$). Hence, during each time period of length τ seconds, flow h gets to flow $x_h C$ bits which is shown to be at least τx_h bits.
Step 2: Let $h \in \bar{\Phi}_f$. We need to show that flow h can be scheduled in the interval $[0, \tau]$. We make the following observation. Flows in $\Phi_h - \Phi_f$ are the only unscheduled flows that contend with h since flows in $\Phi_h \cap \Phi_f$, which also contend with h, are already scheduled in Step 1. In other words, h cannot be active during a time period in S if there exists a flow $g \in \Phi_h - \Phi_f$ which is active during the same period.
Based on the above observation, to show that flow h is schedulable in S, it suffices to show that there exists a time period in S during which all the flows in $\Phi_h - \Phi_f$, including flow h, are schedulable.
Since from Lemma 1, we have $\sum_{g \in \Phi_h - \Phi_f} x_g \leq \sum_{g \in \Phi_f - \Phi_h} x_g$, it follows that all flows in $\Phi_h - \Phi_f$ can be scheduled concurrently with flows in $\Phi_f - \Phi_h$.■

[2] $\mathcal{H} = (F, E)$ is complete if for every pair $(f, g) \in F^2$ there exists an edge $e \in E$ joining f and g. That is, $\forall\, (f, g) \in F^2$, flows f and g contend with each other.



(a) Network graph $\mathcal{G}$.

(b) Contention flow graph $\mathcal{H}$.

(c) Time schedule S.

Fig. 1. Graphs and time schedule for Example 1.

Example *1:* In this example, we show that the sufficient condition given in Proposition 1 is not necessary. Consider the network $\mathcal{G} = (N, F)$ shown in Fig. 1(a) where $N = \{n_1, n_2, n_3, n_4, n_5\}$ and $F = \{f_1, f_2, f_3, f_4\}$. Let us assume IEEE 802.11-based medium access control protocol. That is,

TABLE I

CONTENTION SETS AND WEIGHTS OF FLOWS FOR EXAMPLE 1.

flow f	contention set Φ_f	weight $\delta(f, x)$
f_1	$\{f_1, f_2, f_3\}$	$3 \times \frac{1}{3}C = C$
f_2	$\{f_1, f_2, f_3, f_4\}$	$4 \times \frac{1}{3}C = \frac{4}{3}C$
f_3	$\{f_1, f_2, f_3, f_4\}$	$4 \times \frac{1}{3}C = \frac{4}{3}C$
f_4	$\{f_2, f_3, f_4\}$	$3 \times \frac{1}{3}C = C$

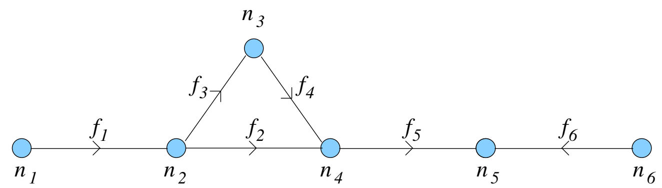
(i) a node can only transmit or receive at any time, and (ii) if node n is transmitting to node m, then all nodes within the same communication range of n or m must remain idle. Fig. 1(b) shows the contention flow graph $\mathcal{H} = (F, E)$. Consider a flow rate vector x equal to $(\frac{1}{3}C, \frac{1}{3}C, \frac{1}{3}C, \frac{1}{3}C)$ (the rate of each flow is $\frac{1}{3}C$ bits per second). The contention sets and weights of flows in F under the flow rate vector x are given in Table I. Note that the flow rate vector x is physically feasible even though the sufficient condition is not satisfied ($\delta(f_2, x) = \delta(f_3, x) = \frac{4}{3}C > C$). Fig. 1(c) shows a feasible time schedule S for the flow rate vector x during which f_1 and f_4 are scheduled simultaneously since they do not contend with each other.■

Corollary *1:* Let $\mathcal{H} = (F, E)$ be complete, and $x = (x_g)_{1 \leq g \leq |F|}$ be a flow rate vector. x is feasible in $\mathcal{H}$ if and only if $\sum_{g \in F} x_g \leq C$.

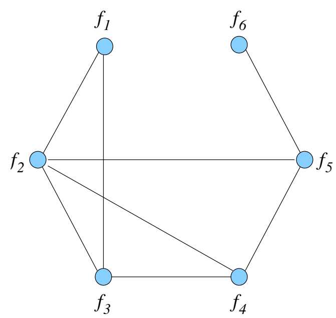
Proof of Corollary 1: Note that $\Phi_f = F$ for all $f \in F$ which implies that $\sum_{g \in F} x_g = \sum_{g \in \Phi_f} x_g$ for any $f \in F$. Thus, the sufficiency of the condition is satisfied from Proposition 1. Now we prove that the condition is necessary. Since x is feasible in $\mathcal{H}$, then there exists a time schedule $S = [0, \tau > 0]$ during which every flow g is active during a period τ_g and flows τx_g bits. Since $\mathcal{H}$ is complete, then $\tau_f \cap \tau_h = \emptyset$ for all $f \neq h$. This implies that $\sum_{g \in F} \tau_g = \tau$. Now since x is feasible, for every $g \in F$, $\tau x_g \leq \tau_g C$. By summing over all flows, we obtain $\tau \sum_{g \in F} x_g \leq \sum_{g \in F} \tau_g C = \tau C$. By simplifying by τ, it follows that $\sum_{g \in F} x_g \leq C$.■

IV. ILLUSTRATIVE EXAMPLES

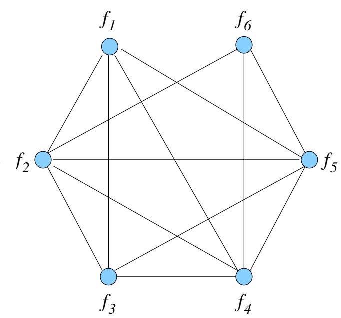
As mentioned earlier in the paper, the feasibility of a flow rate vector depends mainly on (i) the network topology–i.e., how nodes are linked to each other, and (ii) the MAC protocol. For example, in [5], it is assumed that flows which do not share nodes can be active at the same time. That is, the only constraint is that each node can communicate with at most one other node at a time. Systems of this type consist of nodes that, for example, use different frequencies if they are within each other's vicinity. The sufficient condition, in [5], is applicable only for medium access protocols satisfying this requirement. The commonly used IEEE 802.11 MAC protocol does not satisfy this requirement. The condition derived in this paper, however, is applicable for a wider class of MAC protocols, including the IEEE 802.11.



(a) Network graph $\mathcal{G}$.



(b) $\mathcal{H}_1$: MAC of [5].



(c) $\mathcal{H}_2$: MAC of IEEE 802.11.

Fig. 2. Graphs for Example 2.

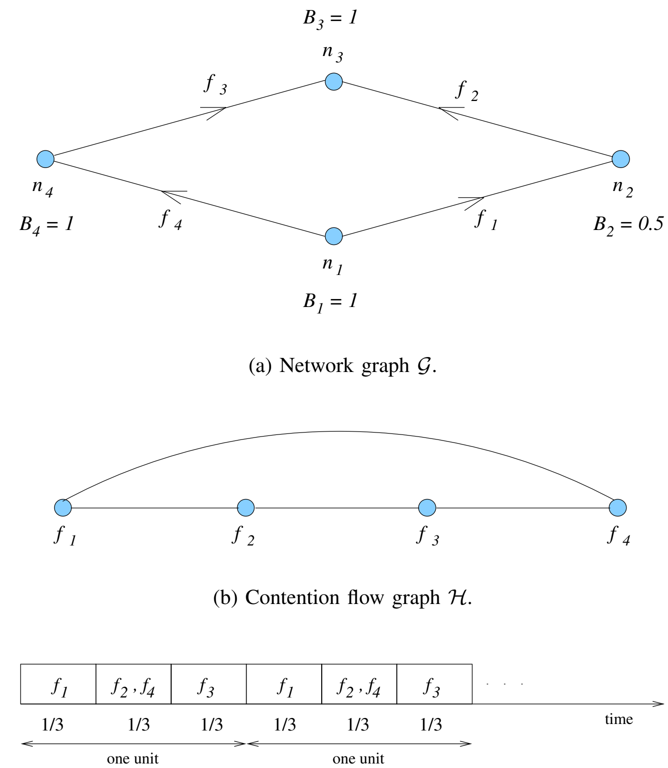
TABLE II

CONTENTION SETS FOR EXAMPLE 2.

flow f	contention set Φ_f: MAC of [5]	contention set Φ_f: MAC of IEEE 802.11
f_1	$\{f_1,f_2,f_3\}$	$\{f_1,f_2,f_3,f_4,f_5\}$
f_2	$\{f_1,f_2,f_3,f_4,f_5\}$	$\{f_1,f_2,f_3,f_4,f_5,f_6\}$
f_3	$\{f_1,f_2,f_3,f_4\}$	$\{f_1,f_2,f_3,f_4,f_5\}$
f_4	$\{f_2,f_3,f_4,f_5\}$	$\{f_1,f_2,f_3,f_4,f_5,f_6\}$
f_5	$\{f_2,f_4,f_5,f_6\}$	$\{f_1,f_2,f_3,f_4,f_5,f_6\}$
f_6	$\{f_5,f_6\}$	$\{f_2,f_4,f_5,f_6\}$

Example *2:* Consider the network $\mathcal{G} = (N, F)$ shown in Fig. 2(a) where $N = \{n_1, n_2, n_3, n_4, n_5, n_6\}$ and $F = \{f_1, f_2, f_3, f_4, f_5, f_6\}$. In this example, we consider two types of MAC protocols: the MAC protocol used in [5], and the MAC protocol of IEEE 802.11. Let $\mathcal{H}_1 = (F, E_1)$ and $\mathcal{H}_2 = (F, E_2)$ denote the contention flow graphs of $\mathcal{G}$ under these two MACs. The graphs $\mathcal{H}_1$ and $\mathcal{H}_2$ are shown respectively in Figs. 2(b) and 2(c). The contention sets of all flows in F are given in Table II. Given a flow rate vector x, by verifying the sufficient condition on each flow using Table II, we can determine the feasibility of x under the MAC of [5] and the MAC of IEEE 802.11.■

Routing solutions are often formulated as linear programming problems (LPPs). For example, in [8], the problem of finding optimal routing solutions that maximize the network lifetime is formulated as a LPP. This formulation, however, does not account for contention constraints resulting from the shared medium. As a result, rate solutions may be infeasible in the sense that the MAC protocol cannot satisfy



(a) Network graph $\mathcal{G}$.

(b) Contention flow graph $\mathcal{H}$.

(c) Time schedule S.

Fig. 3. Graphs and time schedule for Example 3.

the required rates. In the following example, we illustrate the importance of including the medium contention constraints in formulating these routing problems.

Example *3:* Consider a network $\mathcal{G} = (N, F)$ consisting of a set of 4 nodes $N = \{n_1, n_2, n_3, n_4\}$, and a set of 4 flows $F = \{f_1, f_2, f_3, f_4\}$ as shown in Fig. 3(a). Assume that node n_1 cannot directly communicate with node n_3 and node n_2 cannot directly communicate with node n_4. To demonstrate the significance of including the MAC constraints, we solve the same linear programming formulation given in [8] twice: without contention constraints (as defined in [8]), and with contention constraints (by including the sufficient condition on rate feasibility). Let B_i be the initial amount of energy in Joule of n_i's battery. Assume that $B_1 = B_3 = B_4 = 1$ and $B_2 = 0.5$. Let $\epsilon = 0.01$ be the amount of energy in Joule needed to transmit one bit over a flow. Assume that the capacity C of the medium is 1 bit per second.

Now, suppose that the only traffic in the network is from node n_1 to node n_3 at rate of $\frac{2}{3}$ bit per second. The LPP in [8] is to find a flow rate vector $x = (x_g)_{1 \le g \le 4}$ that maximizes the least remaining energy of all nodes over a period T seconds subject to flow balance constraints and energy consumption constraints (refer to [8] for more details). Assume that the period T is 1 second. The flow balance constraints as cited in [8] are:

$$\begin{aligned} 2/3 &= x_1 + x_4, \\ x_1 &= x_2, \\ x_2 + x_3 &= 2/3, \\ x_4 &= x_3, \\ x_g &\ge 0, \quad \forall\, g = 1, 2, 3, \text{ or } 4. \end{aligned}$$

The energy consumption constraints state that the energy consumed by each node after T seconds is less than or equal to the initial amount of available energy.

One can easily verify that this LPP formulation has a unique optimal solution $x = (0, 0, \frac{2}{3}, \frac{2}{3})$. The feasibility of this solution, however, depends on the MAC protocol. For instance, consider the MAC of [5] where each node can communicate with at most one node at a time. Since the MAC protocol of [5] does not allow a node to receive and transmit at the same time, then flows f_3 and f_4 cannot be active simultaneously. As a result, a solution in which flows f_3 and f_4 both flow at a rate greater than the medium capacity of the wireless channel is not physically feasible under this MAC. Hence, the above solution x is not feasible because $x_3 + x_4 = \frac{2}{3} + \frac{2}{3}$ exceeds the capacity of the medium of 1 bit per second.

However, if we include the MAC constraints given by Proposition 1 to the linear programming formulation defined in [8] and solve it, then the resulting optimal solution will be feasible. Fig. 3(b) shows the contention flow graph $\mathcal{H}$ of the network $\mathcal{G}$ under MAC of [5]. Specifically, the constraints given by Proposition 1 are:

$$\begin{aligned} x_4 + x_1 + x_2 &\le 1, \\ x_1 + x_2 + x_3 &\le 1, \\ x_2 + x_3 + x_4 &\le 1, \\ x_3 + x_4 + x_1 &\le 1, \end{aligned}$$

and the optimal solution to the new LPP is $x' = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. Note that, unlike x, x' is now feasible under the MAC of [5]. Fig. 3(c) shows a feasible time schedule S during which flows f_2 and f_4 are scheduled simultaneously.

Through this example, we show the importance of including the MAC contention constraints to routing formulations, and their effect on the feasibility of rate solutions.■

For some MAC protocols, the resulting contention flow graphs are complete. Examples of networks using such MACs are Bluetooth and sensor networks. For these MACs, we can apply Corollary 1 to determine the necessary and sufficient MAC constraints. We illustrate this by the following example.

Example *4:* In this example, we consider the WINS (Wireless Integrated Network Sensors) NG 2.0 platform-based network [1, 3]. The WINS node has the ability to communicate simultaneously on two independent Modems. Each Modem can be independently configured as either *remote* or *base*. A remote Modem must be associated with a base Modem of another node. Each node can be configured so that it can have two concurrent

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flows. Remotes that communicate with the same base constitute a cluster. Neighboring clusters use different radios so that they do not interfere with each other. A cluster can have at most 8 remotes. Within a cluster only one communication can occur at a time. Communication across clusters is allowed only through base nodes. We consider the WINS network given in Fig. 4(a). Let $\mathcal{H} = (F, E)$ be the contention flow graph of the WINS network. Since all flows in cluster 1 contend for accessing the medium of the base of node n_3, and no other flows in the network contend for that medium, the corresponding contention flow subgraph of these flows constitutes a disjoint complete subgraph in $\mathcal{H}$. Similarly, the contention flow subgraphs of all flows in cluster 2 and cluster 3 constitute two other disjoint complete subgraphs in $\mathcal{H}$. Fig. 4(b) shows the three disjoint complete subgraphs constituting $\mathcal{H}$. Therefore, Corollary 1 can be used to impose necessary and sufficient MAC constraints.

In fact, for this MAC protocol, it can be shown that the contention flow graph is always comprised of one or more disjoint complete subgraphs. As a result, one can always use the result of this paper (Corollary 1) to impose necessary and sufficient MAC constraints. We have applied this to solve the data aggregation problem in the SensIT testbed [10].■

V. Conclusion

In this paper, we prove a sufficient condition under which a flow rate vector is feasible given a medium access control (MAC) protocol. We also prove that the sufficient condition is necessary for some MAC protocols such as those used in WINS sensor and Bluetooth networks. Through illustrative examples, we show that the sufficient condition can be used in solving optimization problems (e.g, finding optimal routes) subject to MAC contention constraints. We show that without imposing MAC contention constraints, linear programming-based route solutions may be unfeasible in the sense that the MAC protocol cannot satisfy the rates. A real example of WINS sensor networks is also considered. We show that one can use the results in this paper to impose necessary and sufficient MAC constraints to find routes for networks such as WINS sensor and Bluetooth networks.

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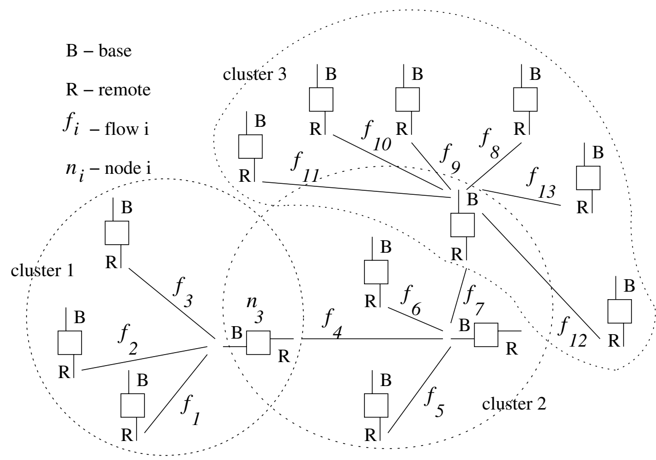
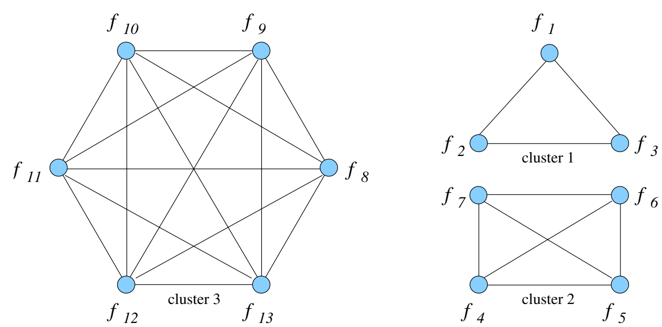


(a) Network graph $\mathcal{G}$.



(b) Contention flow graph $\mathcal{H}$.

Fig. 4. Graphs for Example 4.

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