ECE 331 Introduction to Random Signal Analysis and Statistics

Homework #5 Suggested Solutions

(Updated: 4/9/2001)

Problem , 4-17, 4-26, 4-28, 4-29, 4-31, 4-32, 4-37, 4-38. Chapter 5, 5-4, 5-6

1. (10 points, 4-17) $B_1 = \{x; g(x) \le 1\} = R$, $B_{1/2} = \{x; g(x) \le 1/2\} = [-3/2 \quad 3/2]$, and $B_0 = \{x; g(x) \le 0\} = [-1 \quad 1]$
2. (10 points, 4-26) $F_Y(y) = 1$ for $y \ge 1$, $= 0$ for $y \le 0$, and $= 1-(2/\pi)\cos^{-1}(y)$ for $0 < y < 1$. $f_Y(y) = 2/(\pi\sqrt{1-y^2})$ for $0 \le y < 1$, $= 0$ elsewhere.
3. (10 points, 4-28) Use B given in page 110, example 4-14, we have
   (a) X~uniform[–1, 1]: $F_Y(y) = (y + y^{1/2})/2$ for $0 \le y < 1$, and $= 1$ for $1 \le y$, and 0 for $y < 0$.
   $f_Y(y) = 0.5 + 0.25/y^{1/2}$ for $0 < y < 1$, and $= 0$ elsewhere
   (b) X~uniform[–1, 2]: $F_Y(y) = (y + y^{1/2})/3$ for $0 \le y < 1$, and $= (y+1)/3$ for $1 \le y < 2$, $= 1$ for $2 \le y$, and 0 for $y < 0$.
   $f_Y(y) = 1/3 + 1/(6y^{1/2})\, I_{(0,1)}(y) + (1/3)I_{[1,2)}(y) +$ ~~$(1/3)\,\delta(y-2)$~~
   (c) X~uniform[–2, 3]: $F_Y(y) = (y + y^{1/2})/5$ for $0 \le y < 1$, and $= (y+2)/5$ for $1 \le y < 2$, $= 1$ for $2 \le y$, and 0 for $y < 0$.
   $f_Y(y) = 1/5 + 1/(10y^{1/2})$ for $0 < y < 1$, and $= 1/5$ for $1 \le y < 2$, and $= 0$ elsewhere
   (d) X~exp(λ): $P(a \le X \le b) = e^{-\lambda a} - e^{-\lambda b}$ $(b > a > 0)$. Hence, for $0 \le y < 1$, $F_Y(y) = P(0 \le X \le y) + P(3 \le X) = 1 - e^{-\lambda y} + e^{-3\lambda}$. For $1 \le y < 2$, $F_Y(y) = P(0 \le X \le y) + P(3 \le X) = 1 - e^{-\lambda y} + e^{-3\lambda}$. $F_Y(y) = 0$ for $y < 0$, and $= 1$ for $y \ge 2$. Thus, $f_Y(y) = \lambda e^{-\lambda y}$ for $0 \le y < 2$, and $= 0$ elsewhere.
4. (10 points, 4-29) $B = \varnothing$ for $y < 0$, $= [-y-1 \quad y+1]$ for $0 \le y < 1$, $= R$ for $1 \le y$. Hence
   (a) X~uniform[–1, 1]: $F_Y(y) = I_{[0\ \infty)}(y)$, $f_Y(y) = \delta(y)$
   (b) X~uniform[–2, 2]: $F_Y(y) = (y+1)/2$ for $0 \le y < 1$, $= 0$ for $y < 0$, and $= 1$ for $1 \le y$. $f_Y(y) = 0.5\delta(y) + 0.5I_{(0\ 1)}(y)$
   (c) X~uniform[–3, 3]: $F_Y(y) = (y+1)/3$ for $0 \le y < 1$, $= 0$ for $y < 0$, and $= 1$ for $1 \le y$. $f_Y(y) = (1/3)\delta(y) + (1/3)I_{(0\ 1)}(y) + (1/3)\delta(y-1)$
   (d) X~Laplace(λ): $F_Y(y) = \displaystyle\int_{-y-1}^{y+1} \frac{l}{2} e^{-l|x|}dx = \int_0^{y+1} e^{-lx} d(lx) = 1 - e^{-l(y+1)}$ for $0 \le y < 1$, $= 0$ for $y < 0$, and $= 1$ for $1 \le y$. $f_Y(y) = (1-e^{-\lambda})\delta(y) + \lambda e^{-\lambda(y+1)} I_{[0\ 1]}(y) + e^{-2\lambda}\delta(y-1)$
5. (10 points, 4-31) $F_Y(y) = 0$ for $y < -1$, $= (y+1+\sqrt{y+1})/6$, for $-1 \le y < 1$, $= (3+\sqrt{y+1})/6$ for $1 \le y < 8$, and $= 1$ for $8 \le y$. Thus, $f_Y(y) = 0$ for $y < -1$ or $8 \le y$, $= (1+1/(2\sqrt{y+1}))/6$ for $-1 \le y < 1$, $= 1/6\delta(y-1)$ for $y = 1$, and $= 1/(12\sqrt{y+1})$ for $1 \le y < 8$.
6. (10 points., 4-32) $F_Y(y) = 0$ for $y < 0$, $= 1/4$ for $y = 0$, $= (y + 1 + y^2 + \sqrt{y})/4$ for $0 \le y < 1$, $= 1$ for $1 \le y$. $f_Y(y) = (1/4)\delta(y) + [1+2y+1/(2\sqrt{y})/4]\bullet I_{(0\ 1)}(y)$
7. (10 points, 4-37)

(a) (3 points, CC) $r(t) = (t- 1)^2+1$ *for t* ³ *0.*

(b) (7 points) $\int_0^t (u^2 - 2u + 2)du = t^3/3 - t^2 + 2t$. Hence $f_T(t) = (t^2- 2t+2)exp[- (t^3/3- t^2+2t)]$ for t ≥0.

8. (10 points, 4-38)

(a) $R(t) = I_{[0\ 1)}(t) + (2–x)\ I_{[1\ 2)}(t)$

(b) $r(t) = –R'(t)/R(t) = (1/(2–x))\ I_{[1\ 2)}(t)$

(c) MTTF = $\int_0^\infty R(t)dt = \int_0^1 1dt + \int_1^2 (2-t)dt = 1 + 1/2 = 1.5$

9. (10 points, 5-4) $\lim_{y \to \infty} \frac{e^{-y} - e^{-xy}}{y} = 0$. Hence $F_X(x) = x–1$ for $1 \le x < 2$, = 1 for x > 2 and = 0 otherwise. $F_Y(y) = 1–(e^{–y}–e^{–2y})/y$ for y > 0, = 0 for y ≤ 0.

10. (10 points, 5-6) $f_{XY}(x,y) = [\exp(–x^2/2)/\sqrt{2p}\ ]\bullet\exp(–|y–x|)/2$. However, $\int_{-\infty}^{\infty} \frac{1}{2} e^{-|y-x|} dy = 1$ (Laplace pdf), Hence, $f_X(x) = \exp(–x^2/2)/\sqrt{2p}$ ~ N(0,1).