Department of Electrical and Computer Engineering
University of Wisconsin - Madison

ECE 331 Introduction to Random Signal Analysis and Statistics

Examination #1 Solution

Friday, February 16, 2001, 12:05 – 12:55 PM

1. (20 points)

   (10 points) Simplify $[2 \quad 5] \cap ([1 \quad 3] \cup [4 \quad 6])^c$
   Answer: $[2 \quad 5] \cap ([1 \quad 3] \cup [4 \quad 6])^c = [2 \quad 5] \cap ((-\infty \quad 1) \cup (3 \quad 4) \cup (6 \quad \infty)) =$ **(3 4)**

   (10 points) Simplify $\bigcup_{n=1}^{\infty} [5 \quad 7-(3n)^{-1}]$

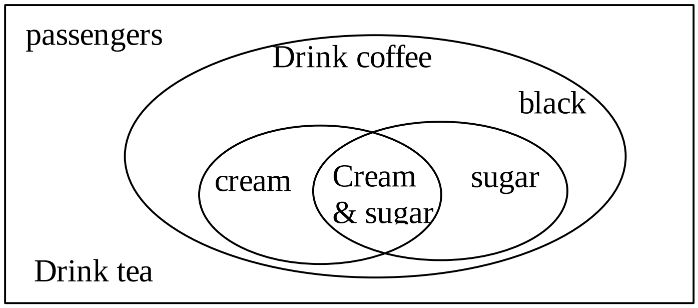
   Answer: $\bigcup_{n=1}^{N} [5 \quad 7-(3n)^{-1}] = [5 \quad 7-(3N)^{-1}]$. Thus, $\lim_{N\to\infty} [5 \quad 7-\frac{1}{3N}]$ = **[5 7)**

2. (25 points)
   During the flight from Madison to Minneapolis, coffee and tea are served. A passenger must choose either coffee or tea. If a passenger chooses coffee, he or she must decide one of four possible flavors: (i) black (no cream and no sugar), (ii) cream only, (iii) sugar only, and (iv) cream and sugar.

   (a) (10 points) Suppose that the probability a passenger chooses coffee is 0.4. *Given* that a passenger wants coffee, the probability that this person wants black coffee is 0.3. What is the probability that an arbitrary passenger in this flight wants a black coffee?
   Answer: Denote event C = *a passenger chooses coffee*, and event B = chooses black coffee. Then P(C) = 0.4, and P(B|C) = P(B∩C)/P(C) = 0.3. An arbitrary passenger chooses black coffee = B∩C. Hence **P(BÇ C) = P(B|C)P(C) = 0.3*0.4 = 0.12**.

   (b) (10 points) Continue from part (a). Suppose that the probability a passenger chooses coffee and wants to add sugar to it (choice of sugar only or cream-and-sugar) is 0.24; and that the probability a passenger chooses coffee and wants to add cream to it (choice of cream only or cream-and-sugar) is 0.16. What is the probability that a passenger wants *both* cream and sugar in the coffee *given* that this passenger wants a cup of coffee?
   Answer: Denote S = add sugar, and R = add cream. We have P(C∩S) = 0.24, P(C∩R) = 0.16. We want to find P(S∩R|C). Note that P(S|C) = P(C∩S)/P(C) = 0.24/0.4 = 0.6, and P(R|C) = P(C∩R)/P(C) = 0.16/0.4 = 0.4. Also, P(B|C) = 1 – P(S∪R|C) = 0.3. Hence, P(S∪R|C) = 0.7 = P(S|C) + P(R|C) – P(S ∩R|C) = 0.6 + 0.4 – P(S∩R|C). Therefore, **P(SÇ R|C) = 1 - 0.7 = 0.3**.

   (c) (5 points) Let the *universe* of a Venn diagram be all the passengers on a flight. Plot a Venn diagram depicting passengers not drinking coffee as well as drinking coffee with the four different ways.
   Answer:



3. (30 points)
The University Department of Information Technology has n dial-up lines to serve remote access to campus network by students and faculty. The probability that a dial-up line is busy is independent and identically distributed (i.i.d.) among all n dial-up lines. Suppose that on a Friday night, the probability that a dial-up line is busy is 0.7

(a) (10 points) What is the probability that <u>all</u> dial-up lines are busy when you dial in?
Answer: Define $X_i = 1$ if a dial up line is busy. Then it is a Bernoulli random variable with parameter $p = 0.7$. Thus, $P(X_i = 1) = 0.7$, and $P(X_i = 0) = 0.3$. Probability of all dial-up lines are busy is
$\mathbf{P(X_1 = 1, X_2 = 1, \ldots, X_n = 1) = p^n = 0.7^n}$

(b) (10 points) A switching computer will find an idle dial-up line when you dial in whenever there is one or more lines available. What is the probability that when you dial in, you can connect to one of the n dial up line?
Answer: Note that unless all dial-up lines are busy, you can always connect to one of the dial-up line. Thus, the answer is $\mathbf{1 - p^n = 1 - 0.7^n}$.

(c) (10 points) What is the probability that exactly *k* ($0 \le k \le n$) dial-up lines are idle?
Answer: $\binom{n}{k} 0.3^k 0.7^{n-k}$; $0 \le k \le n$.

4. (25 points)
Dr. Who taught the probability course several times. He found that 75% of students who did homework pass the exam; while only 15% of students who did not do homework pass the exam.
(a) (15 points) If 80% of students do the homework in a class, what percentages of students pass the exam?
Answer: $P(pass|H) = 0.75$, and $P(pass|H^c) = 0.15$. $P(H) = 0.8$, and $P(H^c) = 1–0.8 = 0.2$. Hence, $P(pass \cap H) = P(pass|H)P(H) = 0.75*0.8 = 0.6$, $P(pass \cap H^c) = P(pass|H^c)P(H^c) = 0.15*0.2 = 0.03$. Use total probability, **P(pass) = P(passÇ H) + P(passÇ $\mathbf{H^c}$) = 0.6 + 0.03 = 0.63.**
(b) (10 points) Of those students who pass the exam, what percentages did homework?
Answer: P(H|pass) = P(HÇ pass)/P(pass) = 0.6/0.63 = 20/21 = 0.9524.