ECE334 HOMEWORK 8

1 Suppose that $\dot{x} = Ax + bu$ and $y = cx$ where

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \text{and} \qquad b = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \qquad \text{and} \qquad c = (1 \;\; 0)$$

(a) Design a state feedback K to place both poles at $-0.5 \pm j0.5$. Check that your answer works.
(b) Design an observer gain L so that both observer poles are at $-1 \pm j$. Check that your answer works.
(c) Write down the symbolic equations of the plant compensated with a dynamic observer (use the standard symbols $A, b, c, K, L, x, \hat{x}$). Draw a block diagram of the system.
(d) Write down the equations of the plant compensated with a dynamic observer using numerical values. Confirm that the eigenvalues of the plant compensated with a dynamic observer are what the theory says they are.
(e) Derive from the equations in (c) the equations of the plant compensated with a dynamic observer in terms of the vector $b = \begin{pmatrix} x \\ e \end{pmatrix}$ where $e = x - \hat{x}$. State the separation principle.

2 Consider the 2 carts connected by a spring and driven by 2 motors in homework 5 problem 5. Suppose that we measure the position of cart 1 so that the single output $y = z_1$. Design the gains L for an observer so that the observer poles are in a fourth-order Butterworth pattern of radius 1; that is, the characteristic equation becomes

$$s^4 + 2.613s^3 + (2 + \sqrt{2})s^2 + 2.613s + 1 = 0$$

You may recall from earlier homeworks that for this system with state $x = (z_1, z_2, \dot{z}_1, \dot{z}_2)^T$ and the given numerical values of parameters,

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -Kr^2R\alpha & Kr^2R\alpha & -k^2\alpha & 0 \\ Kr^2R\alpha & -Kr^2R\alpha & 0 & -k^2\alpha \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1.0 \\ -0.016 & 0.016 & -0.04 & 0 \\ 0.016 & -0.016 & 0 & -0.04 \end{pmatrix}$$